

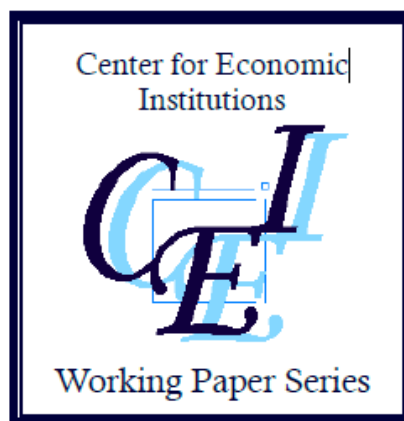
Center for Economic Institutions
Working Paper Series

No. 2025-3

**“What’s in A Price? – Valuation for Welfare
And National Accounts”**

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March, 2026



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What's in A Price? - Valuation for Welfare and National Accounts

Draft version 1.3

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April 14, 2026

Abstract

Modern economies combine market and non-market products and producers. Although many market products have observable prices, many digital services are offered at zero monetary cost, relying instead on data capture or advertising, and their consumer value is insufficiently captured in national accounts. Public services face a similar issue, as they are typically valued at production cost rather than by the consumer welfare they generate. Recent advances in measurement — such as incentivized online experiments — offer a way to quantify welfare effects. However, not every welfare valuation is suitable for comparison or integration into the National Accounts. We develop a unified framework for valuing market and non-market products, introduce virtual and reservation prices, average compensation, unit costs, quality-adjustment of prices, and clarify their relationship to the national accounts and price index measurement.

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1 Introduction

In modern economies, households consume a wide range of goods and services, from traditional market goods—such as cars and groceries—to digital services, public healthcare, education or defence. While typical market products are exchanged at observable prices, many essential services are provided for free or at nominal fees. Such services may or may not be supplied by market providers.

Digital services, in particular, are now often offered to consumers 'for free', meaning there is no explicit market transaction. Instead, transactions are implicit: consumers agree to share personal data or view advertisements. The value of these transactions is hidden within the value of final products. While this value is not lost in national accounts—it contributes to the market value of the advertised products—it does not necessarily reflect the welfare values that 'free' products offer to consumers.¹

Similarly, government-provided services such as free or subsidised health care, education, housing or defence enter the national accounts with the costs that it takes to provide them but not necessarily with the value that they provide to citizens.

Recently, methodological advances have permitted quantifying such welfare valuations contributions, starting with free digital products² and expanding to other goods and services, witness the GDP-B ('Benefits') project at Stanford University's Digital Economy Lab³.

The core methodology involves capturing consumers' willingness to accept

¹See Nakamura, Samuels and Soloveichik[32] or Bourgeois[8] for the valuation of free digital products and Ahmad, Ribarsky and Reinsdorf[1] for measuring the digital economy more generally.

²For instance, Brynjolfsson, Collis, Diewert, Eggers, and Fox (BCDEF)[7] developed metrics to measure the welfare gains from using services like Facebook and smartphone cameras. Brynjolfsson, Collis, Liaqat, Kutzman, Garro, Deisenroth, Wernerfelt, and Lee[6] have estimated the welfare effects of 10 popular digital goods across 13 countries. Coyle and Nguyen[9] have measured the WTA for free digital products in the U.K..

³See <https://digitaleconomy.stanford.edu/gdp-b/>.

compensation for giving up a product (or consume less of a product) they do have. This is done through large-scale, incentivized online choice experiments with representative samples. For government services, additional avenues can be taken to measure their welfare effects. Quality adjustment to medical care prices or quantities using life years gained is a case in point.

This text uses a unified framework for valuing both market and non-market goods within a household's consumption bundle. At its core is the conditional expenditure function, which defines the minimum cost required for a household to achieve a given utility level, given market prices and the availability of free or subsidized goods. This function provides the foundation for exploring key valuation concepts, including market prices, virtual prices, average compensation, reservation prices, unit costs (quasi-prices), and quality-adjusted prices.

The analysis begins with *market prices*, which are observable and reflect both the marginal utility of consumption and the marginal cost of production. From a consumer perspective, for goods and services provided for free or at reduced fees—such as digital apps, public healthcare, or education—*virtual prices* (or marginal compensation prices) are essential. These prices represent the implicit value consumers place on non-market goods, even in the absence of explicit transactions. Virtual prices enable the integration of such goods into welfare and price index measurements, effectively bridging the gap between market and non-market valuation. As conceptual counterparts to market prices, they can be consistently compared and combined.

Reservation prices, a special case of virtual prices, address the challenge of valuing new products before they enter the market and after they exit the market. By capturing the marginal welfare effects of new product introductions, they ensure consistency with price index theory and national accounts valuation.

Average compensation, in contrast, measures the total monetary amount required to induce a consumer to accept a reduced quantity—or even forgo entirely—a free or subsidized product. This concept is relevant for captur-

ing full welfare effects and results are not in general comparable to market valuations or virtual prices that rest on marginal valuation concepts.

From the producer's perspective, *unit costs* (quasi-prices) offer a method for valuing non-market goods, such as government-provided services. These costs approximate the value of goods when market prices are absent, though they may not align with consumer valuations.

The conditional expenditure function is also a tool to frame quality adjustment of prices. Once more, the importance of marginal, rather than average, valuations becomes apparent when accounting for changes in product quality, such as in healthcare or education.

Finally, we briefly address *accounting prices*, which broaden the scope of partial and static analysis to include inter-temporal and social welfare dimensions. By incorporating natural, human, and social capital into comprehensive capital valuation, accounting prices offer a framework for evaluating the long-term sustainability of socio-environmental-economic systems. However, we also argue that their practical implementation poses significant challenges, often requiring a balance between conceptual ideals and real-world feasibility.

Our *tour d'horizon* of prices is limited to private effects. With the exception of the short digression on accounting prices, it does *not* provide a valuation framework for social effects that would capture negative and positive externalities from consuming or producing the products under consideration.

2 The framework

We consider N 'normal' market products $q \equiv [q_1, \dots, q_N] \geq 0_N$ that a household purchases at market prices $p \equiv [p_1, \dots, p_N] \gg 0_N$, such as a car or a kilogramme of apples. Additionally, we consider M goods or services $z \equiv [z_1, \dots, z_M] \geq 0_M$ that are provided for free (no explicit monetary transaction) or at a nominal fee $w_z \equiv [w_{z_1}, \dots, w_{z_M}] \geq 0$. These could include digital services like smartphone apps (social media, navigation, etc.) provided by private suppliers, as well as government services (such as public health care)

offered either free of charge or against the subsidized fee w_z . z could also stand for quality characteristics of products, a case examined at greater detail below.

Household utility is represented by a utility function $u = u(q, z)$ where u is non-negative, non-decreasing in its arguments, quasi-concave and continuous in q and z ⁴. The conditional expenditure function⁵ of the household is given by⁶:

$$E(u, p, w_z, z) = \min_q \{p \cdot q + w_z \cdot z : u(q, z) \geq u\}. \quad (1)$$

This function $E(u, p, z)$ indicates the minimum expenditure required to achieve utility level u , given a set of market prices p , exogenous ('environmental') variables z , provided at $w_z \geq 0$. Equation (1) will constitute the reference point for our taxonomy of price and valuation concepts.

2.1 Market prices

Concept. This is the most straight forward price and valuation concept: there are explicit and observable transactions. Market prices correspond to unit values of market products at a detailed level that, multiplied with the corresponding quantities, give rise to observed expenditure for a particular product, $p^t \cdot q^t$. Cost minimising choices of market products, given free or subsidised products z , and market prices p imply that the observed expenditure in period t equals minimum cost: $E(u^t, p^t, w_z^t, z^t) = p^t \cdot q^t + w_z^t \cdot z^t$. These are the nominal values that are typically recorded in the System of National Accounts (SNA). To describe the nature of market prices and how

⁴Our analysis considers a single representative consumer only. It is not difficult to see, however, that - as soon as a household-specific utility function is introduced or different households benefit from different quantities of z - virtual prices become household-dependent or dependent on different reference values for z and u . This is not the case for market prices, considered exogenous to households.

⁵See Browning[3] for the properties of the conditional expenditure function.

⁶We use the notation $\sum_{i=1} p_i q_i \equiv p \cdot q$.

they compare with other approaches towards valuation, note the following well-known characteristics.

- First, a marginal rise in the price of market product i will lead to an extra expenditure that equals the quantity of product i in the cost-minimizing consumption bundle (Shephard's[38] Lemma):

$$\frac{\partial E(u, p, w_z, z)}{\partial p_i} = q_i \quad (i = 1, \dots, N). \quad (2)$$

Equation 2 defines the Hicksian market demand function $q_i(u, p, z)$. We shall use this relationship later for the discussion of welfare values and the measurement of consumer surplus. It further holds that⁷

$$\frac{\partial E(u, p, w_z, z)}{\partial w_{z_j}} = z_j \quad (j = 1, \dots, M) \quad (3)$$

Despite the similarity with (2), (3) does not define a demand function for z since z is not freely chosen. Instead, it captures the marginal cost exposure to changes in w_{z_j} , holding z , u and p fixed.

- Second, for cost-minimising consumers, a market price p_i equals the money value of the marginal utility from consuming an additional unit of q_i : $p_i = \frac{\partial E(u, p, w_z, z)}{\partial u} \frac{\partial u(q, z)}{\partial q_i}$ ⁸. In a competitive market, prices will also equal the marginal costs of production with unit or average costs as a special case - see Section 2.6. Hence, money-valued marginal utility from consuming a product equals marginal costs of producing it.

Conclusion. None of the above is new but worth re-stating to conclude that the standard approach towards valuing transactions with market prices

⁷This is due to the fact that $q_i(u, p, z)$ does not depend on w_z . The derivative of $E(u, p, w_z, z) = p \cdot q(u, p, z) + w_z \cdot z$ with respect to w_{z_j} is therefore z_j .

⁸The marginal utility of money, λ , is defined as $\frac{\partial E(u, p, w_z, z)}{\partial u} = \lambda$. The Lagrange function for cost minimisation is $L = p \cdot q + w_z \cdot z + \lambda [u - u(q, z)]$. Then $\frac{\partial L}{\partial u} = \lambda$. If quantities q are cost-minimising, it follows from the Envelope Theorem that $\frac{\partial E(u, p, w_z, z)}{\partial u} = \frac{\partial L}{\partial u} = \lambda$. It also holds that $p_i = \lambda \frac{\partial u(q, z)}{\partial q_i}$. Then $p_i = \frac{\partial E(u, p, w_z, z)}{\partial u} \frac{\partial u(q, z)}{\partial q_i}$: in equilibrium, a price p_i is equal to the money value of the marginal utility derived from q_i , given z .

is *marginal* in nature - both from the consumer's and from the producer's perspective.

2.2 Virtual prices or marginal compensation prices

Concept. Virtual prices represent the unit value which would cause a consumer at u and facing market prices p to choose z_j . Virtual prices are relevant for products or services provided either for free or at a nominal fee (although it is often the case with 'free' digital products that personal data is bartered implicitly for access to an application). Suppliers of z_j may be private entities - think of corporations providing digital applications - or public entities such as government units that provide health, education or defence services at a nominal fee w_z . We shall label virtual prices as $w_z^s \equiv [w_{z_1}^s, \dots, w_{z_M}^s]$. For a cost-minimising consumer, $w_{z_j}^s$ is defined through

$$\frac{\partial E(u, p, w_z, z)}{\partial z_j} \equiv -w_{z_j}^s \leq 0 \quad (j = 1, \dots, M). \quad (4)$$

Although w_z^s are often referred to as 'shadow prices', we follow Coyle and Wdowin[10], and reserve this notion to the valuation of *social* benefits, including possible externalities. While w_z^s may not be observable through explicit monetary transactions, it is a *private*, not a social valuation in nature. As such, virtual prices resemble market prices p conceptually and can be compared consistently. They both stand for private exchange values. Yet another way of expressing their communality is that both virtual and market prices reflect the marginal willingness to give up or to pay for an extra unit of z_j or a unit of q_i , given a level of utility u .

Suppose that w_z^s can be measured with some empirical technique. Then, the valuations for market products q and 'free' products z can be combined through an unconditional expenditure function $C(u, p, w_z^s)$ for total minimum

costs, defined as (see BCDEF[7]):

$$\begin{aligned}
C(u, p, w_z^s) &\equiv \min_{q,z} [p \cdot q + w_z^s \cdot z : u(q, z) \geq 0] \\
&= \min_z [\min_q (p \cdot q + w_z \cdot z + (w_z^s - w_z) \cdot z : u(q, z) \geq 0)] \\
&= \min_z [\min_q (p \cdot q + w_z \cdot z : u(q, z) \geq 0) + (w_z^s - w_z) \cdot z] \\
&= \min_z [E(u, p, w_z, z) + (w_z^s - w_z) \cdot z], \tag{5}
\end{aligned}$$

where the last line of (5) defines again the virtual prices as in (4). If (q, z) is a solution to the cost minimisation in (5), it must hold that $C(u, p, w_z^s) = p \cdot q + w_z^s \cdot z$. We then obtain an exact relationship between the unconditional and the conditional expenditure function (Deaton[15]):

$$C(u, p, w_z^s) = E(u, p, w_z, z) + (w_z^s - w_z) \cdot z = p \cdot q + w_z^s \cdot z. \tag{6}$$

When the elements of $p \cdot q + w_z^s \cdot z$ are known, standard index number theory can be applied to measure price and volume changes of an extended consumption aggregate that comprises both traditional market consumption and the consumption of free products.

Application. When market producers offer goods or services for free or at reduced nominal fees, virtual or marginal compensation prices provide the conceptually appropriate way to value such products when the goal is to compare them with market-based values, such as those recorded in the national accounts. A similar argument applies to free government social transfers in kind, including healthcare, education, or housing. Several qualifications apply, though.

Accounting issues. An independent valuation of free services using virtual prices—where such prices exist—raises questions about how to integrate these valuations into the System of National Accounts. Because the system is based on double (or quadruple) entry accounting, any imputed value for a product must have a corresponding entry in production or imports. It is therefore useful to distinguish between private suppliers of free products and government suppliers.

- Applying virtual prices to free products supplied by private corporations—most commonly free digital services—would require imputing the same value to corporate profits or to imports. Schreyer[35] argues that such imputations should not be recorded in the corporate sector but in the household sector. There, virtual prices can be interpreted as the unit value of services that households obtain from using digital products as part of their own-account household production. For instance, households may use free apps and their own time to produce entertainment services for themselves. These services would then be valued using virtual prices rather than the nominal transaction value of the app.
- For government services provided free of charge, valuing them using virtual prices would imply departing from the principle that non-market services should be valued as the sum of the costs required to produce them. In this case, the sum of costs would differ from the value of the government service, generating an imputed positive or negative surplus for government that is difficult to interpret. One possible solution is to constrain the total nominal value of government services to equal their costs while using virtual prices only to weight individual services when deriving volume measures of government service — see Section 2.6.

Estimating virtual prices. A further issue concerns the practical measurement of virtual prices. What is observed are nominal fees w_z —possibly zero—which may or may not correspond to virtual prices w_z^s . Again, it is helpful to distinguish two cases.

- First, consider a situation in which nominal fees w_z^t in period t are regarded as approximations of virtual prices $w_z^{s,t}$. In this case, the observed quantities z^t can be interpreted as the outcome of an unconditional cost-minimisation problem in which q^t and z^t are freely chosen given p^t and w_z^t . Total expenditure based on exchange values can then be measured directly as a special case of equation (5), namely

$p^t \cdot q^t + w_z^t \cdot z^t$. As will be shown below, in some cases the assumption $w_z^{s,t} = w_z^t$ may be necessary to derive reservation prices when only nominal fees w_z^t are observable.

- Second, consider the situation $w_z^t \neq w_z^{s,t}$, where there is no good reason to assume that virtual prices coincide with nominal fees. For example, the observed fee may be zero, as is the case for public safety or defence services. The consumer's virtual price will often be positive and therefore differ from the fee. In such cases, an independent estimate of $w_z^{s,t}$ is required. Approximations can sometimes be derived from information on the *average compensation* (see Section 2.3) that a consumer would need to receive to forgo a certain quantity of products, though this introduces additional simplifying assumptions into the model.

In the general case where nominal fees differ from virtual prices, total expenditure equals the part of the consumer's budget chosen freely plus expenditure on z , as captured by the conditional expenditure function:

$$E(u, p^t, w_z^t, z^t) = p^t \cdot q^t + w_z^t \cdot z^t. \quad (7)$$

This is indeed what is recorded in the national accounts, but it risks understating money-metric utility because the term $(w_z^{s,t} - w_z^t) \cdot z^t$ is omitted - see equation (6). Public healthcare provides an example. The quantity of medical care is not in general a matter of personal choice but a necessity. And although z_j represents the quantity of medical care provided by public healthcare at the rate $w_{z_j} \geq 0$, a consumer's willingness to pay for additional care may be higher. The variable z_j could also reflect the quality of medical care, a case discussed further in Section 2.7 on quality adjustment.

Conclusion. Under a consumer perspective, and with a view to comparing results with the National Accounts, virtual prices, or marginal compensation prices, are the correct concept for valuing goods and services provided at

nominal or zero fees. By defining the unit value that aligns consumer choices with utility maximization, virtual prices bridge the gap between market-based and non-market valuations. Virtual prices constitute private exchange values analogous to market prices.

However, their practical application, in particular their integration into the National Accounts, raises challenges. For private suppliers, they may lead to artificially overstating corporate profits and for government services, they may conflict with cost-based valuation principles.

2.3 Average compensation

Concept. Average compensation refers to the monetary amount required to induce a consumer to accept a (significantly) smaller quantity of a free product or to forego it entirely. Taking product z_1 as an example, let $\tilde{z}_1 \geq 0$ denote the reduced quantity, which may be zero. The consumer is then asked to value the loss of $z_1 - \tilde{z}_1 \geq 0$. Further, let $z_{-1}^t \equiv [z_2^t, \dots, z_M^t]$ denote the quantities of all other products that are free or sold for a nominal fee. The compensation with regard to product 1, $m_{z_1}^t$, required by the consumer can be expressed as

$$\begin{aligned} E(u^t, p^t, w_z^t, \tilde{z}_1, z_{-1}^t) - E(u^t, p^t, w_z^t, z^t) &= m_{z_1}^t \\ E(u^t, p^t, w_z^t, \tilde{z}_1, z_{w_z^t, -1}^t) - [p^t \cdot q^t - w_z^t \cdot z^t] &= m_{z_1}^t. \end{aligned} \quad (8)$$

$E(u^t, p^t, w_z^t, \tilde{z}_1, z_{-1}^t)$ corresponds to Diewert and Mizobuchi's[22] version of Samuelson's[33] concept of money-metric utility or Fleurbaey's[26] [27]'s equivalent income. $m_{z_1}^t$ is a measure of compensating valuation, or the income needed to make the consumer indifferent between enjoying \tilde{z}_1 and z_1^t , everything else being equal.

Fleurbaey[26] highlights the advantages of welfare comparisons based on equivalent income or money metric utility, particularly the possibility of making interpersonal welfare comparisons by adjusting for differences in individual circumstances and preferences. More generally, the framework accommodates non-monetary aspects of well-being, including free products.

Application. A recent and compelling application of the concept of average compensation is found in the choice experiments conducted by researchers at the Stanford Digital Economy Lab⁹ - see Brynjolfsson, Collis and Eggers[5] or Brynjolfsson, Collis, Diewert, Eggers and Fox[7] - who evaluate the monetary compensation required for consumers to forego free digital products. In our framework, this corresponds to measuring $m_{z_1}^t$. Dividing $m_{z_1}^t$ by $z_1^t - \tilde{z}_1^t$ yields the *average compensation* required to give up $z_1^t - \tilde{z}_1^t$ units of product 1, $w_{z_1}^{a,t} \equiv m_{z_1}^t / (z_1^t - \tilde{z}_1^t)$. In the special case where the consumer gives up the entire quantity of z_1^t so that $\tilde{z}_1^t = 0$, the resulting average compensation is $w_{z_1}^{a,t} = m_{z_1}^t / z_1^t$. This corresponds to the results from the Stanford choice experiments for free products and so provides a theoretical foundation for welfare measurement using the choice-experiment approach. Bourgois[8] discusses different approaches towards valuing free digital products and also concludes that the choice experiments put forward by Brynjolfsson, Collis and Eggers[5] ”...seem to be the clearest in terms of the objectives pursued.” (p.165)

The average valuation $w_{z_j}^{a,t}$ does not, in general, coincide with the marginal valuation $w_{z_j}^{s,t}$ ($j = 1, \dots, M$). Each serves a distinct analytical purpose. Nevertheless, average valuations can provide a useful starting point for inferring marginal values, with reservation prices being a case in point as discussed in Section 2.4 below.

A different application of average compensation measures evaluating the benefits of public services to the consumers, coupled with several theoretical advances, is Schwartz, Diewert and Fox[37] who present methods and data requirements involved in assessing consumer benefits from infrastructure services. They then extend the analysis to account for the different periods over which the infrastructure change occurs, thereby allowing p^t to vary. They further show how welfare effects for heterogeneous households can be aggregated to regional totals.

Conclusion. The concept of average compensation provides a coher-

⁹<https://digitaleconomy.stanford.edu/>

ent way to assign monetary values to free or non-priced goods by linking changes in quantities to the income needed to preserve consumer well-being. By grounding choice-experiment evidence in a formal welfare framework, it strengthens the measurement of benefits from digital products, public services, and other non-market goods. By assessing total welfare effects, this approach complements marginal valuation methods and supports broader welfare comparisons across individuals and contexts.

2.4 Reservation prices - a special case of virtual prices

Concept. Diewert, Fox and Schreyer[21] introduce the rationale for reservation prices as follows: "A major problem facing statistical agencies is how to adjust household price and quantity indexes for increases in the choice of commodities. The main concept for dealing with this problem is to use a framework suggested by Hicks (1940), where it is assumed that households have (latent) preferences defined over products before they appear in the marketplace. If reservation prices for these unavailable products can be estimated for the period before their introduction to the market place, then normal index number theory, based on the economic approach to index numbers, can be applied." (p. 895)

The Hicksian reservation price is the price at which a utility-maximising consumer would choose to demand zero units of a product. Before a new product is introduced to the market, its demand is necessarily zero, which is equivalent to saying that its market price would have to match its reservation price. The gap between this reservation price and the first price at which demand becomes positive reflects the (often overlooked) initial decline in prices that accompanies the introduction of new products. In terms of our workhorse, the conditional expenditure function and its derivative regarding different elements of z in (4), a reservation price for product 1 in period t is defined as

$$\frac{\partial E(u, p, w_z, 0, z_{-1})}{\partial z_1} \equiv -w_{z_1}^r, \quad (9)$$

or the virtual price at which demand for z_1 drops to zero. Now consider again a choice experiment as described earlier that reveals the average compensation needed to forego consumption of product 1 in period t , $w_{z_1}^{a,t} \equiv m_{z_1}^t / z_1^t$. Following the Diewert, Fox and Schreyer[21] procedure and forming two first-order Taylor series approximations to $m_{z_1}^t$:

$$\begin{aligned}
E(u^t, p^t, w_z^t, 0, z_{-1}^t) &\approx E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t) + \frac{\partial E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t)}{\partial z_1} [0 - z_1^t] \\
&= E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t) - w_{z_1}^{s,t} [0 - z_1^t] \text{ (using (4))} \\
&= E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t) + w_{z_1}^{s,t} z_1^t \\
E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t) &\approx E(u^t, p^t, w_z^t, 0, z_{-1}^t) + \frac{\partial E(u^t, p^t, w_z^t, 0, z_{-1}^t)}{\partial z_1} [z_1^t - 0] \\
&= E(u^t, p^t, w_z^t, 0, z_{-1}^t) - w_{z_1}^{r,t} [z_1^t - 0] \text{ (using (9))} \\
&= E(u^t, p^t, w_z^t, 0, z_{-1}^t) - w_{z_1}^{r,t} z_1^t. \tag{10}
\end{aligned}$$

There are now two equally valid approximations to the difference $E(u^t, p^t, w_z^t, 0, z_{-1}^t) - E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t)$ and we form the following arithmetic average¹⁰

$$\begin{aligned}
E(u^t, p^t, w_z^t, 0, z_{-1}^t) - E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t) &\approx \frac{1}{2} (w_{z_1}^{s,t} + w_{z_1}^{r,t}) z_1^t = m_{z_1}^t \\
\text{and further } m_{z_1}^t / z_1^t = w_{z_1}^{a,t} &\approx \frac{1}{2} (w_{z_1}^{s,t} + w_{z_1}^{r,t}) \text{ or} \\
w_{z_1}^{r,t} &\approx 2w_{z_1}^{a,t} - w_{z_1}^{s,t}. \tag{11}
\end{aligned}$$

The final step is evaluating the reservation price for a period $\tau < t$ prior to the availability of the product on the market at t . As Diewert, Fox and Schreyer[21] show, this is achieved by deflating the period t reservation price $w_{z_1}^{r,t}$ by the average price change of the continuing products between τ and t .

Application. The relation in (11) goes some way towards measuring a reservation price but one unknown remains, the marginal valuation or virtual price $w_{z_1}^{s,t}$. A first option is approximating $w_{z_1}^{s,t}$ by $w_{z_1}^{a,t}$. In this case, the

¹⁰The approximation will be an exact one if $E(u, w_z, p, z)$ is a quadratic function of z_1 between 0 and z_1^t ; see the quadratic approximation lemma in Diewert[20].

reservation price would be $w_{z_1}^{r,t} \approx 2w_{z_1}^{a,t} - w_{z_1}^{a,t} = w_{z_1}^{a,t}$. There may also be cases of 'lumpy' products that can either be used or not and consumers are asked to value a zero-one choice. One example is BCDEF's[7] estimation of the value of consumers' access to Facebook which is either on or off. For an individual consumer, the marginal valuation does not exist (or coincides with the average valuation). In this case, again, the reservation price at t approximately equals $w_{z_1}^{a,t}$.

Given that the conditional expenditure function $E(u, p, w_z, z)$ is non-increasing and convex in z for fixed (u, p, w_z) under regular conditions, the average valuation at t , $w_{z_1}^{a,t} = (E(u^t, p^t, w_z^t, 0, z_{-1}^t) - E(u^t, p^t, w_z^t, z_1^t, z_{-1}^t)) / z_1^t$, will exceed the marginal valuation $w_{z_1}^{s,t}$ at z_1^t . The above approximation will then understate the reservation price and, by implication, overstate inflation¹¹.

Alternatively, if the observed nominal fees are considered good approximations to the virtual prices ($w_{z_1}^{s,t} \approx w_{z_1}^t$), the reservation price in (11) is $w_{z_1}^{r,t} = 2w_{z_1}^{a,t} + w_{z_1}^t$. This is the avenue chosen by Diewert, Fox and Schreyer[21].

Weale[42] uses a reservation price approach to capture the effect of new medical treatments on the price index of medical care. Whereas Diewert, Fox and Schreyer[21] rely on choice experiments to first obtain an average compensation and then a reservation price, Weale uses evidence on the gains in quality adjusted life years (QALYs) due to the new treatment to estimate a reservation price. The money value of the gain in QALYs is estimated by applying the Value of a Statistical Life Year - a constant measure per year so average and marginal valuation coincide. It could also be reasoned that receiving or not receiving a treatment is a zero-one situation from the perspective of a single person so the distinction between average and marginal utility of the gain in QALYs is not defined in such a case.

Conclusion. We conclude that reservation prices, as a special case of

¹¹The expenditure function would have to be linear in z for marginal and average valuation to coincide. Note that even if the consumer's utility function is linear in z , for example $u(q, z) = v(q) + \alpha z$, the conditional expenditure function becomes $E(u, p, w_z, z) = E(u - \alpha z, p, w_z)$ and is not linear in z .

virtual prices, can be consistently derived from observed transactions and choice experiments. They can then be integrated into price or volume indexes that are conceptually consistent with national accounts data using standard index number theory.

2.5 Virtual prices, average compensation and consumer surplus - a digression

Concept. In 1844, Dupuit[24] suggested that the area under the demand curve and above the price represents the 'surplus utility' enjoyed by the consumer. This concept of consumer surplus measures the additional utility of those consumers that would have been willing to pay a higher price than the prevailing market price. It is depicted as the area under a demand curve above the prevailing market price. Traditionally, changes in consumer surplus have been used to measure welfare changes between different market equilibria.

However, the original definition of consumer surplus relies on the Marshallian (uncompensated) demand function where quantities depend on income and other prices. These are generally observable. Several authors have noted¹² that interpreting changes in consumer surplus as welfare change requires a constant marginal utility of income which is a restrictive assumption.

To address this issue, Hicks[28] developed Equivalent and Compensating Variations as surplus measures. These are based on compensated demand functions where quantities depend on prices and utility levels. The average compensation measure in equation (8) is directly related to the Hicksian surplus measures.

To show this relation, consider first the inverse demand function inherent in the definition (4) of the virtual price for product z_j where we take again product z_1 as an example: $w_{z_1}^s = w_{z_1}^s(u, p, w_{z_1}, z_1, z_{-1})$ which shows the money value that consumers are willing to accept for giving up one unit of z_1 . The

¹²See Slesnick[40] for a review and discussion.

area under the inverse demand function between quantities z_1^t and \tilde{z}_1 is

$$\begin{aligned} \int_{\tilde{z}_1}^{z_1^t} w_{z_1}^s(u, p, w_z, z_1, z_{-1}) dz_1 &= \int_{\tilde{z}_1}^{z_1^t} -\frac{\partial E(u, p, w_z, z_1, z_{-1})}{\partial z_1} dz_1 \\ &= E(u, p, w_z, \tilde{z}_1, z_{-1}) - E(u, p, w_z, z_1^t, z_{-1}) \end{aligned}$$

and for realised prices and quantities in period t,

$$E(u^t, p^t, w_z^t, \tilde{z}_1^t, z_{-1}^t) - E(u^t, p^t, w_z^t, z_1^t) = m_{z_1}^t, \quad (12)$$

where $m_{z_1}^t$ is again the required compensation for consuming z_1 in smaller quantities ($\tilde{z}_1^t < z_1^t$). This corresponds to the general form of a Hicksian consumer surplus measure (see Slesnick[40]).

Conclusion. We conclude that the welfare measures derived through choice experiments are closely linked to Hicksian surplus measures. Hicksian surplus measures, based on compensated demand functions are more appealing theoretically than Marshallian surplus measures. However, compensated demand functions are typically hard to estimate empirically, which is why welfare analysis often relies on uncompensated demand functions and Marshallian consumer surplus. Choice experiments can overcome this empirical challenge by providing estimates based on compensated demand functions, thereby marking an important advancement.

2.6 Unit costs or quasi prices - the producer perspective

Concept. When goods or services are provided by a non-market producer such as the government, the prices charged to consumers are typically significantly below the actual unit cost of production. In some cases, no price is charged at all. These prices reflect administrative or policy choices rather than the economic value of the products to either consumers or producers.

One way to address this issue is to value such products on the basis of unit production costs. Koechlin, Konjin, Luca and Schreyer[29] refer to these as *quasi-prices*: unobserved “prices” that approximate a competitive market

situation in which prices equal average costs per unit of output. In this approach, unit costs are treated as if they were market prices (Diewert[17],[16]; Schreyer[36]).

Unlike the valuation approaches discussed in the preceding sections, valuation using quasi-prices is grounded in the producer perspective. This may differ from the consumer perspective, except in the case of market prices where the two perspectives are assumed to coincide (see Section 2.1).

More specifically, we assume that all of the items in z are produced although this could easily be expanded to leave out some elements that are also exogenous to the producer side. The production process of z can be characterised by a technology set F^t such that in period t the set of products z is producible with inputs x : $(z, x) \in F^t$.¹³ Production for period t can then be characterised by the following cost function:

$$\phi^t(z, w_x) = \min_x \{w_x \cdot x \mid (x, z) \in F^t\}, \quad (13)$$

where $\phi^t(z, w_x)$ depicts the minimum cost of producing $z \equiv [z_1, \dots, z_M]$ different types of goods or services and $x \equiv [x_1, \dots, x_K]$ is the vector of inputs (labour services, capital services, intermediate inputs) required to produce z . Inputs are purchased at prices $w_x \equiv [w_{x_1}, w_{x_2}, \dots, w_{x_K}]$. We further assume that producers minimise costs so that in any period t actual costs equal minimum costs: $\phi^t(z^t, w_z^t) = w_z^t \cdot x^t$. Cost-minimising behaviour also implies that $\frac{\partial \phi^t(z^t, w_x^t)}{\partial w_{z_k}} = x_k^t$ ($k = 1, \dots, K$) (Shepard[38]). We take it that input prices and quantities can be observed.

Marginal costs of producing output z_j^t in period t are $\frac{\partial \phi^t(z^t, w_x^t)}{\partial z_j} \geq 0$ but it is not clear that these can actually be observed with non-market producers in the absence of market prices. In some cases, it may be possible to find close substitutes supplied by market producers whose prices could be imputed to value z .¹⁴ More often in practice, it will only be possible to estimate unit costs or quasi prices for each product. This requires knowledge about the

¹³For a general characterisation of production and cost functions see Diewert[18][19] or Sickles and Zelenyuk[39], for an application to health care services, see Schreyer

¹⁴This is Diewert's[16] Case 2 (p.179) in his hierarchy of methods to value non-market

cost allocation across products - an entirely empirical matter. Suppose that $\phi_j^t(z^t, w_x^t)$ is the cost allocated to z_j in period t . It follows that

$$\phi^t(z^t, w_x^t) = \sum_j^M \phi_j^t(z^t, w_x^t) = \sum_j^M p_{z_j}^{q,t} z_j^t = w_x^t \cdot x^t$$

where $p_{z_j}^{q,t} = \frac{\phi_j^t(z^t, w_x^t)}{z_j^t}$. (14)

The terms $p_{z_j}^{q,t}$ are the unit costs or quasi prices for z_j in period t . If the production technology exhibits constant returns to scale such that the cost function is linear homogenous in z ¹⁵, we can apply Euler's Theorem to obtain

$$\phi^t(z^t, w_z^t) = \sum_j^K \frac{\partial \phi^t(z^t, w_z^t)}{\partial z_j} z_j^t. \quad (15)$$

In this case, cost allocation based on marginal and average costs coincides, and unit costs equal marginal costs for each z_j . In the general case of non-constant returns to case, however, there is no straight forward relationship between product-specific marginal and average costs.

We note that although unit costs or quasi prices may deviate from marginal costs, the overall nominal value of output of non-market production V^t in period t is measured as $V^t = \sum_{j=1}^J p_{z_j}^{q,t} z_j^t = w_x^t \cdot z^t$, which is equal to the sum of costs.

Application. The national accountant's task of measuring production and quasi prices begins with identifying the units that produce the relevant services. For non-market producers, the value of gross output during a particular period is measured by adding the following costs incurred in production:

- intermediate consumption (the goods and services used up in producing the service),

production. Diewert provides a full treatment of cost-based measures of outputs, inputs, and productivity under constant returns to scale in production, and the associated index numbers.

¹⁵More precisely, $\phi^t(\lambda z, w_z) = \lambda \phi^t(z, w_z)$ for all $\lambda > 0$.

- compensation of employees (wage and salaries of personnel including employer contributions, etc.),
- consumption of fixed capital (depreciation of buildings, of equipment, etc.),
- other taxes, less subsidies, on production.

A notable absence in non-market production, compared to market producers, is the profit margin. This is inherent to the nature of non-market production, where residual profits do not exist by definition. Historically, national accounts rules only accounted for capital costs through depreciation, overlooking other capital-related costs. However, the 2025 System of National Accounts[25] has addressed this asymmetry by introducing imputations for a 'net rate of return' - a rate that reflects financing costs for non-market producers that would be added to the above.

Once total costs for non-market production are determined, they must be allocated across different products to establish quasi prices. Such cost allocations are increasingly common in essential public services like health and education, where cost accounting plays a pivotal role in service management and cost control.

Schreyer[34] highlights that the accuracy of administrative cost allocations as proxies for true average costs remains difficult to verify. However, if cost-allocation processes involve well-informed stakeholders - for example in healthcare - and since the primary goal of health care cost accounting in most countries is to capture the full costs of treatments accurately, significant deviations between quasi prices and true average costs seem unlikely. Similar practical challenges arise in education.

For nearly all goods and services—whether provided by market or non-market producers—quality adjustments for volumes or prices remain a critical consideration. This is addressed in the following section.

Conclusion. Non-market goods and services can be valued using unit production costs, treated as quasi-prices that stand in for the missing market

prices. These quasi-prices reflect the producer’s cost structure for each type of product produced rather than consumers’ willingness to pay. In practice, total production costs—labour, intermediate inputs, capital consumption, taxes, and an imputed cost of capital—are allocated across outputs to derive these quasi-prices. They equal average costs rather than marginal costs although it may be reasonable - or indispensable given available data - to assume that production exhibits constant returns to scale in which case the difference disappears. Their accuracy depends heavily on the quality of administrative cost-allocation systems, especially in sectors like health and education. Valuation of non-market services with quasi prices is fully compatible with national accounts practice.

2.7 Quality-adjusted prices

When the characteristics of goods or services change, when new product varieties appear—expanding consumers’ choices—or when entirely novel products enter the market, a key question emerges: how should prices and price indexes be adjusted for quality change? This issue arises for all types of “prices” mentioned earlier, whether market, virtual, or quasi-prices; whether they relate to goods or services; whether they are produced by market or government providers; and whether the index concerns consumer or producer prices.

The scope of the question is therefore broad and the discussion here is selective¹⁶: we focus on a method where quality adjustments are implemented by capturing the welfare gains (or losses) from consuming a product. A prominent domain in which such direct welfare adjustments have been applied is health care - see for example Cutler, McClellan, Newhouse, Remler[13] Cutler, Rosen, Vijan[12], Dunn, Hall, Dauda[23], and Weale[42]. In this section, we:

- show how the standard approach in the literature for capturing welfare

¹⁶For an overview of quality adjustment methods in the context of consumer prices see Chapter 7 of the Consumer Price Index Manual[31]

gains from improved products - specifically, medical procedures - can be formulated consistently using the conditional expenditure function (13), which has served as a unifying thread throughout our discussion of prices;

- address the question whether or not welfare-based quality adjustments are suitable for national accounts.

Concepts. For clarity, we expand the utility function to include an element $h(z_1)$ where h represents an outcome variable—such as an individual’s health status or human capital. Health outcomes might be quantified using metrics like quality-adjusted life years, while educational outcomes could be assessed through discounted lifetime earnings. This outcome, $h(z_1)$, depends on one or more external elements in the cost function. We interpret these elements as quality attributes of one of the market product q - for example, the time saved by self-booking a flight - or of a non-market product such as the quality of medical care or the time parents devote to assisting their children with schoolwork. For simplicity, we assume that the nominal fees w_z equal zero but the extension to a non-negative value is straight forward.

The utility function thus takes the form

$$v = v [q, h(z_1), z_{-1}] \quad (16)$$

where $h(z_1)$ is taken as differentiable and non-decreasing in z_1 . As earlier, $z_{-1} \equiv [z_2, \dots, z_M]$ captures all other 'environmental' variables that affect utility.

The conditional expenditure function in (1) becomes

$$\tilde{E}(u, p, z) = \min_q \{p \cdot q : v [q, h(z_1), z_{-1}] \geq u\}. \quad (17)$$

For cost-minimising consumers, the observed expenditure in period t on market products equals minimum expenditure, $\tilde{E}(u, p^t, z_1^t, z_{-1}^t) = p^t \cdot q^t$. The Konüs[30] cost-of-living index P between periods $t = 0, 1$ which accounts for changes in quality attributes from z_1^0 to z_1^1 is defined as

$$P \equiv \frac{\tilde{E}(u, p^1, z_1^1, z_{-1}^1)}{\tilde{E}(u, p^0, z_1^0, z_{-1}^0)}. \quad (18)$$

This quality-adjusted price change measures the minimum expenditure required in period 1 - given prices p^1 and quality attributes z_1^1 - to maintain the consumer's initial utility level u . All other exogenous variables z_{-1} are held constant at a reference level. Choosing $t = 0$ as the reference period yields the Laspeyres version of P , P_L :

$$P_L \equiv \frac{\tilde{E}(u^0, p^1, z_1^1, z_{-1}^0)}{\tilde{E}(u^0, p^0, z_1^0, z_{-1}^0)}. \quad (19)$$

While the numerator in (19) equals observed expenditure in period 0, the denominator is not observable and must be approximated. A first-order Taylor expansion gives

$$\begin{aligned} E(u^0, p^1, z_1^1, z_{-1}^0) &\approx E(u^0, p^0, z_1^0, z_{-1}^0) + \sum_{i=1} \frac{\partial E(u^0, p^0, z_1^0, z_{-1}^0)}{\partial p_i} [p_i^1 - p_i^0] \\ &\quad + \frac{\partial E(u^0, p^0, z_1^0, z_{-1}^0)}{\partial z_1} [z_1^1 - z_1^0] \\ &= p^0 \cdot q^0 + q^0 \cdot [p^1 - p^0] - w_{z_1}^{s,0} [z_1^1 - z_1^0] \\ &= p^1 \cdot q^0 - w_{z_1}^{s,0} [z_1^1 - z_1^0]. \end{aligned} \quad (20)$$

Substituting this expression into (19) yields

$$\begin{aligned} P_L &\approx \frac{p^1 \cdot q^0 - w_{z_1}^{s,0} [z_1^1 - z_1^0]}{\tilde{E}(u^0, p^0, z_1^0, z_{-1}^0)} = \frac{p^1 \cdot q^0 - w_{z_1}^{s,0} [z_1^1 - z_1^0]}{p^0 \cdot q^0} \\ &= \frac{p^1 \cdot q^0}{p^0 \cdot q^0} - \frac{w_{z_1}^{s,0} [z_1^1 - z_1^0]}{p^0 \cdot q^0}. \end{aligned} \quad (21)$$

Expression (21) indicates that the conventional Laspeyres price index is adjusted for changes in the quality attribute z_1 . Because the adjustment enters with a negative sign, an improvement in quality $[z_1^1 - z_1^0] > 0$

lowers the price index. The quality change valued using the virtual price $w_{z_1}^{s,0} = w_{z_1}^{s,0}(u^0, p^0, z_1^0, z_{-1}^0)$, i.e., the *marginal* willingness to pay for a one-unit increase in the quality attribute at the initial period- 0 situation.

Proceeding analogously—now using period 1 reference values for u and z_{-1} —we could also derive a Paasche-type approximation to the cost-of-living index in (18). In this case, the quality adjustment enters the price index through the term $w_{z_1}^{s,1}[z_1^1 - z_1^0]$ where the virtual price $w_{z_1}^{s,1} = w_{z_1}^{s,1}(u^1, p^1, z_1^1, z_{-1}^1)$ represents the *marginal* willingness to pay for a one-unit increase in the quality attribute evaluated at the comparison period 1. The Laspeyres and Paasche approach could then be averaged as none is inherently preferable.

The main point to be made here, however, is the need to value the change in quality attributes *at the margin* (whether assessed at period 0 or period 1), and *not as an average* valuation of the entire availability of z_1^t , $w_z^{a,t}$ in our notation earlier. Take the quality of medical care as an example - almost certainly, the initial welfare gains from improving on very low medical care quality would be very large whereas subsequent marginal gains would be smaller. Hence, average valuations of quality improvements when quality of medical care is already at a high level would overstate the marginal valuation and thus over-state quality adjustments. This point has been made repeatedly in the literature on health care valuation - see Dunn, Hall, and Dauda[23] - but applies more generally.

As a final conceptual consideration, we provide additional contents to the interpretation of the virtual price w_z^s . The price of a market product p_i and the virtual price of the environmental variable z_1 , defined as $\frac{\partial \tilde{E}(u,p,z)}{\partial z_1} = w_{z_1}^s$, can also be expressed as

$$\begin{aligned} p_i &= \lambda \frac{\partial v[q, h(z_1), z_{-1}]}{\partial q_i} \text{ and} \\ w_{z_1}^s &= \lambda \frac{\partial v[q, h(z), z_{-1}]}{\partial h} \frac{\partial h(z_1)}{\partial z_1}, \end{aligned} \quad (22)$$

where the Lagrange multiplier λ is the marginal utility of expenditure¹⁷.

¹⁷The Lagrange function is $L = p \cdot q + w_z^s \cdot z - \lambda \{v[q, h(z_1), z_{-1}] - Y\}$ where $Y = p \cdot q$

The virtual price $w_{z_1}^s$ measures the marginal effect of a change in z_1 (for example the quality of health care) on health, weighted by the change in utility due to a change in health and expressed in money terms. Solving (22) for λ yields an expression for the relative price $w_{z_1}^s/p_i$ which is exactly the coefficient found in the health care literature by which health care quality improvements are weighted.

Applications. One question here is the extent to which such explicit quality adjustment of prices is a case for the National Accounts. Ackley, Dunn, Liebman, and Romley[2] who apply the above methodology to U.S. medical care prices note "The [...] measure of productivity presented here is based on utility theory and consumer welfare, and is distinct from methods applied elsewhere in the accounts [...]. The focus on welfare measurement more closely aligns with the GDP-B approach of Brynjolfsson et al. (2019). This approach departs somewhat from the SNA 2025 that focuses on the measurement of outputs from economic activity rather than outcomes. However, the distinction between output and outcomes blurs in the health sector as the quality-adjusted price and output depend on the expected outcome of treatments, although not (as a conceptual matter) outcomes due to non-medical factors."

To this should be added that the valuation of the quality change is at the margin, thereby at least in principle compatible with other virtual and market prices. Note that the quality adjustment in (21) can also be expressed in more familiar terms (to price statisticians) as a multiplicative adjustment to price or volume indexes. Take the second line of (21) and apply the logarithmic mean $l(z_1^1 - z_1^0) = \frac{z_1^1 - z_1^0}{\ln(z_1^1/z_1^0)}$:

is the budget constraint and the first order conditions in (22) follow.

$$\begin{aligned}
P_L &\approx \frac{p^1 \cdot q^0}{p^0 \cdot q^0} - \frac{w_{z_1}^{s,0}[z_1^1 - z_1^0]}{p^0 \cdot q^0} \\
&= \sum_{i=1}^N \frac{p_i^0 q_i^0}{p^0 \cdot q^0} \frac{p_i^1 - p_i^0}{p_i^0} + 1 - \frac{w_{z_1}^{s,0}[z_1^1 - z_1^0]}{p^0 \cdot q^0} \\
&= \sum_{i=1}^N \frac{l(p_i^0, p_i^1) q_i^0}{p^0 \cdot q^0} \ln \frac{p_i^1}{p_i^0} + 1 - \frac{l(z_1^1, z_1^0) w_{z_1}^{s,0} \ln \frac{z_1^1}{z_1^0}}{p^0 \cdot q^0} \\
&= \ln P_{\mathcal{L}} - \alpha_z^0 \ln \frac{z_1^1}{z_1^0} + 1 \text{ where} \\
\ln P_{\mathcal{L}} &\equiv \sum_{i=1}^N \frac{l(p_i^0, p_i^1) q_i^0}{p^0 \cdot q^0} \ln \frac{p_i^1}{p_i^0} \text{ and } \alpha_z^0 \equiv \frac{l(z_1^1, z_1^0) w_{z_1}^{s,0}}{p^0 \cdot q^0}. \text{ Then,} \\
P_L - 1 &\approx \ln(P_L) = \ln P_{\mathcal{L}} - \alpha_z^0 \ln \frac{z_1^1}{z_1^0}. \tag{23}
\end{aligned}$$

The last line of (23) indicates that the standard Laspeyres price index (proxied with $P_{\mathcal{L}}$) is adjusted multiplicatively by the growth rate of the quality adjustment variable $\ln \frac{z_1^1}{z_1^0}$. The latter is scaled by α_z^0 , the average marginal valuation of z_1 normalised with market expenditure $p^0 \cdot q^0$. As before, a similar computation could be put in place for the Paasche perspective. The multiplicative quality adjustment has also seen to be relevant when cost-based rather than welfare-based quality criteria are invoked. Examples include the hedonic approaches by Diewert[16] or Schreyer[36] in the case of non-market production¹⁸.

Conclusion. Accurately capturing welfare gains or losses when product characteristics or quality attributes change is important, including in sectors

¹⁸Cutler et al.[11] apply quality adjustments to different medical treatments and use shares in total QALYs gained due to medical care as weights. Alternatively, shares of production costs for different activities could be used. As Weale[42] notes: "The use of cost weights rather than QALY weights has the consequence that the magnitude of the overall quality adjustment will depend on the way in which the individual components are aggregated. If adjustment takes place at the aggregate level and QALYs are the only indicator of quality, then the reported absolute increase in output will be the same as that proposed by Cutler et al.[11]." (p.6)

like healthcare. The key insight here is that quality adjustments—if they are to be conceptually consistent with national accounts—must be valued at the margin, not as average valuations. This distinction is critical: marginal valuations reflect incremental benefits in quality, whereas average valuations risk overstatement, especially when baseline quality is already high.

2.8 Accounting prices or social welfare valuations

Accounting prices are mentioned here for completeness rather than as an extension or modification of our conditional expenditure function that represents a partial consumer equilibrium approach. The literature around accounting prices - well presented in its most comprehensive version by Dasgupta[14] - has two main characteristics:

- It is inter-temporal in nature, featuring a social welfare function that captures the discounted flow of current and future utility such as $U^0 = U(u^1, u^2, u^3, \dots)$, derived from consumption of (q) in each period. Consumption is produced by using labour and capital in conjunction with current and future production technology. The other inter-temporal feature is investment that links current output and expenditure to the evolution of the capital stock, along with depreciation and depletion.
- Capital measures K are assumed comprehensive, encompassing produced and non-produced assets including natural capital, human capital or social capital - in short all types of capital that shape society's well-being. As usual, capital stocks rise with positive investment and decline with depreciation or depletion, thereby linking stocks in each periods to flows.

Dasgupta[14] and Arrow et al[4] introduce the notion of resource allocation mechanisms, i.e., future paths of the economic-environmental-social system, and demonstrate that, in principle, it is possible, by recursive reasoning, to map future flows of consumption and investment onto today's capital stocks

so that $U^0 = U^0(q^0, K^0, z^0, \alpha^0)$ where α^0 is the particular resource allocation mechanism chosen for such a mapping. No optimal behaviour is required to introduce this concept.

Accounting prices are then the marginal social valuations of capital stocks, defined as $p_{AKi} \equiv \frac{\partial U^0(q^0, K^0, \alpha^0)}{\partial K_i}$. The changes in stocks today, each valued at accounting prices, then gauge the change in inter temporal social welfare, depending on whether $\Delta U^0 \gtrless 0$ (Dasgupta[14]).

$$\Delta U^0 = \sum p_{AKi} \Delta K_i^0. \quad (24)$$

A negative sign in the change of comprehensive capital would then indicate a situation non-sustainability of human activity. Tracking changes in the asset base using accounting prices would thus seem to be the most important effort to pursue. But it also turns out to be the most challenging venture, in concept and in practice. There are at least two difficulties here¹⁹.

- Although 'only' present changes in assets need to be observed, their valuation with accounting prices requires projections of the future evolution of the socio-economic-environmental system, because the resource allocation mechanism α has to be described and evaluated. Conceptually, accounting prices reflect all the externalities associated with economic activities, missing markets, or occurrence of "tipping points" in natural assets. Here we are in a different world from that normally inhabited by statistical offices - a world of scenario-building, horizon scanning and comprehensive modelling and forecasting. Resource requirements apart, this raises some important institutional issues.
- The notion of comprehensive wealth encompasses a broad set of assets, from produced machinery to human capital, social capital and natural assets. But what is the exact scope and how should it be measured?

¹⁹See Fleurbaey and Blanchet[27] for a broader discussion of measurement and theoretical questions.

There are many borderline cases, and measurement issues abound, including:

- Whether or not health ought to be recognised as a separate asset, and in addition to human capital is a matter of debate and makes a tremendous difference to results[4].
- Social capital is an asset considered significant for the functioning of societies, i.e., the social norms, shared values and institutional arrangements that foster co-operation among population groups and the trust people have in others and it would appear very difficult to develop corresponding accounting prices.
- Another key question is capturing and valuing ecosystem assets. Their deterioration or improvement represents a big part of what constitutes today’s environmental concerns. But ecosystem assets are complex, dynamic and do not observe national boundaries. Again, modelling accounting prices that attach to the change in ecosystem assets would appear to be a tall order.

Conclusion. The comprehensive social valuation of economic, environmental and social assets is an excellent reference framework to reason about sustainability. But its empirical implementation with an ambition of providing a comprehensive, single indicator of sustainability based on accounting prices raises more questions than it may answer. A pragmatic approach is called for: a fuller implementation of SNA assets, including land, improved physical measures of natural assets, spatially-differentiated valuations of ecosystem services are good places to start. The SEEA[41] provides excellent guidance here.

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