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Forecasting life expectancy: Evidence from a new survival function

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Abstract

We propose a new survival function to forecast life expectancies at various ages. The proposed model comprises the youth-to-adulthood component and the old-to-oldest-old component. It is able to closely fit adult survivorship of the US men and women in the period from 1950 to 2010. We find evidence that the forecasting performance of life expectancies by the proposed model compares favorably with those obtained from the popular Lee-Carter model (1992) and the shifting logistic model proposed by Bongaarts (2005).

Keywords: Lee-Carter model, Life expectancy, Mortality, Survival probability

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Forecasting life expectancy: Evidence from a new survival function

1. Introduction

The importance of modeling human mortality has been widely acknowledged by researchers and practitioners. Actuaries employ mortality rates to price actuarially-fair insurance products while demographers use such information to project future populations, which in turn serves as input to social planning (Yaari (1965), Sheshinski (2007)). More recently, economists have begun to analyze demographic impacts on macroeconomic variables such as capital accumulation and economic growth (Bruce and Turnvosky (2013)). Over the years, there has been a vast literature of various approaches to modeling mortality rates, each having its own merits and weaknesses. See, for example, Gompertz (1825), Makeham (1860), Lee and Carter (1992), Bongaarts (2005) and Booth (2006). On the one hand, while the well-documented Gompertz-Makeham model and the Shifting Logistic model by Bongaarts are able to capture the mortality rates with relatively few parameters, they do not produce satisfactory forecasts of life expectancies at young ages. On the other hand, even though the Lee-Carter approach is able to forecast life expectancies at different ages reasonably well over the short to medium terms, the model is not designed to capture changes in age-specific mortalities. Hence, it may not be suitable for making projections, especially at old ages over time.

In this paper, we propose a new survival function to model adult mortality and to forecast life expectancies at various ages. The proposed model, in the form of a modified exponential-type function, consists of two components: the youth-to-adulthood component and the old-to-oldest-old component. The youth-to-adulthood component captures a constant and low level survival probability from birth to adulthood, which starts declining linearly and slowly from age 35 to 75. The old-to-oldest-old component also captures a constant but high survival probability from birth to age around 50, but starts declining exponentially around age 55 to 110. The parameters can be estimated by the method of non-linear least squares estimation. We find evidence that the proposed model is able to closely fit adult mortality of the US men and women from sample data of life tables for each year covering the period from 1950 to 2010.

Our study follows similar approaches by Bongaarts (2005) and McNown and Rogers (1989), which couple the parameterized model with time series methods to obtain forecasts of age-specific survival probability and mortality. Essentially this tractable two-step approach treats estimates of parameters of the proposed model from each year over a period of time as a set of observations on each parameter. Such time series of parameter estimates can then be modeled and forecasted by the well-documented autoregressive and moving average (ARMA) models to capture the possible trends over time. The forecasted parametric values are then plugged into the survival function to produce forecasts of age-specific mortality. By applying standard time series tests to each of the annual series of parameter estimates generated from the US data of life tables, we find support that lower-order AR models are sufficient to accommodate the dynamic structures of the parameters in the proposed survival function. Moreover, when we engage the proposed model to obtain within-sample and holdout-sample forecasts of life expectancies at different ages from birth to 80, we find that such forecasts compare favorably with those obtained by the popular Lee-Carter model (1992) and the shifting logistic model proposed by Bongaarts (2005).

This rest of this paper is organized as follows. In the next section we highlight the gist of the proposed survival function. In Section 3 we report estimation results of fitting the proposed model to sample data of life tables for the US population by gender and the corresponding time series models of estimates of parameters. In Section 4 we compare the performance of the proposed model by forecasts of life expectancies with the actual ones and with those obtained by the Lee-Carter model and the shifting logistic model by Bongaarts. In Section 5, we report the simulated confidence interval for forecasts of life expectancies and discuss implications of the forecasted decennial survival distributions of the US population by the proposed model. Section 6 provides some concluding remarks.

2. The proposed function

Many studies of mortality rates often focus on modeling the force of mortality of adults. This is partly because such rates can be roughly approximated by exponential-type functions. The Gompertz-Makeham model and the logistic function proposed by Bongaarts are two of the most popular models and have the following specifications:

Gompertz-Makeham: $$\mu(x) = \alpha + \beta e^{\gamma x} \quad (1)$$

Bongaarts: $$\mu(x) = \alpha + \frac{\beta e^{\gamma x}}{1 + \beta e^{\gamma x}} \quad (2)$$

It is well-documented that the Gompertz-Makeham model provides a good fit to adult mortality rates. However, this model tends to over-estimate mortality rates at ages above 80. In contrast, Bongaarts' logistic model is able to address such an insufficiency. A closer inspection of the two functions specified in equations (1)-(2) reveals that if $\beta e^{\gamma x}$ is small, the logistic function is similar to the Gompertz-Makeham function. At the oldest-old ages, however, the logistic function caps the force of mortality at $1+\alpha$ whereas the Gompertz-Makeham function allows it to grow continuously. As such, the logistic function appears to be more robust in approximating adult mortality.

In this study we follow the convention of engaging exponential-type functions in the literature of actuarial science and demography. Instead of modeling the instantaneous death rates, we propose a new survival function to capture the survival probabilities. There are two good reasons for doing so. First, the survival probabilities often exhibit simpler patterns than those of mortality rates, making it more feasible to fit the entire survival function parametrically. Second, when making forecasts of life expectancies, it would be more convenient for practitioners to obtain the age-specific survival probabilities directly without first converting the estimated mortality rates into survival rates. The proposed survival function is specified as follows:

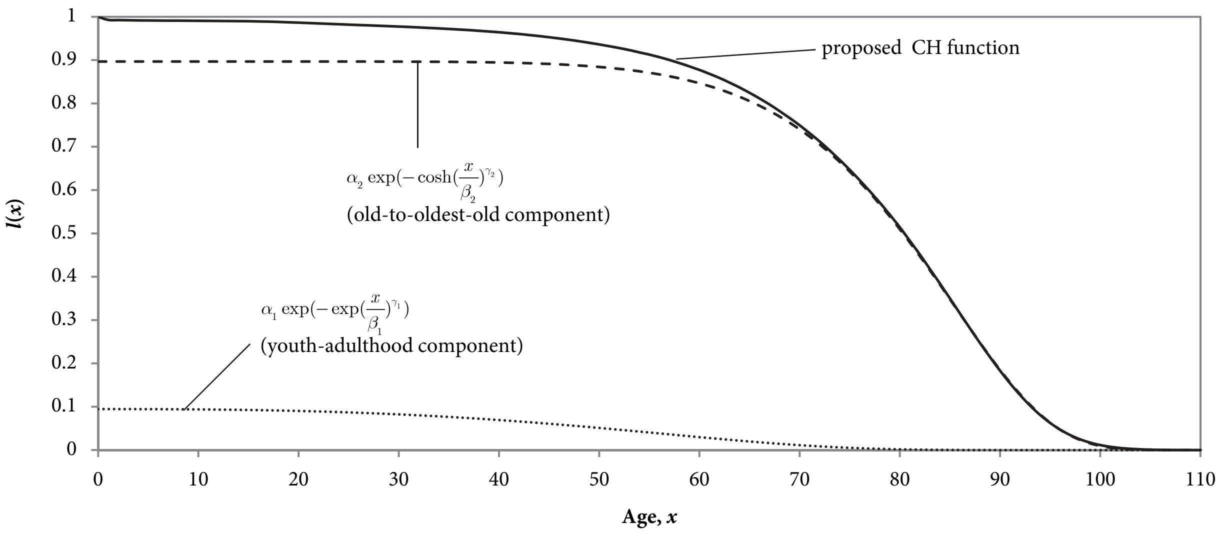
$$S(x) = \alpha_1 \exp(-\exp(\frac{x}{\beta_1})^{\gamma_1}) + \alpha_2 \exp(-\cosh(\frac{x}{\beta_2})^{\gamma_2}),$$

where $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2 \in \mathbb{R}_+$ (3)

Note that the *α's* in equation (3) are the scale parameters which control the weights of the first and second terms on the right side of the equation; the *β*'s, on the other hand, are the horizontal scale parameters; and the *γ's* are the shape parameters. For easy reference, we denote the proposed survival function as the CH function.

Intuitively the CH function can be viewed as a linear combination of two age-specific components. The first component resembles an exponential survival curve while the second component involves the hyperbolic cosine (cosh) function, the even part of e^x. Figure 1 plots the model estimates of the CH function and its two components, based on parameters estimated

Figure 1: Components of the proposed CH function for the combined US population in 2000



from the combined US population in the year 2000. As can be observed, the first term of the equation (3) is estimated to be 0.1 at birth, and remains roughly the same until around age 35. Thereafter, it starts declining very slowly and eventually converges to close to 0 around the age of 75. Since this pattern accounts for the bulk of the variations in human mortality below the age of 55, we interpret it to be the youth-adulthood component of the CH function. In contrast, the second term on the right side of the CH function begins with an estimated value of about 0.9 at age 0, and stays roughly constant until about age 55. It starts to decline exponentially around ages 55 to 110. This mimics the decline in survival rates above the age of 55, or the old to oldest-old ages and is hence named old-to-oldest-old component in the above figure.

As will be shown in the next section, the CH function is able to closely fit adult mortality of the US men and women using life tables covering the period from 1950 to 2010.

We note in passing that the CH function satisfies the following general properties of a survival function:

[a] $S(0) = 1$;

[b] $S(\infty) = 0$;

[c] $S'(x) < 0$, with

$$S'(x) = -\left[\frac{\alpha_1\gamma_1}{\beta_1}\left(\frac{x}{\beta_1}\right)^{\gamma_1-1} e^{-\exp(\frac{x}{\beta_1})^{\gamma_1}} e^{(\frac{x}{\beta_1})^{\gamma_1}} + \frac{\alpha_2\gamma_2}{\beta_2}\left(\frac{x}{\beta_2}\right)^{\gamma_2-1} e^{-\cosh(\frac{x}{\beta_2})^{\gamma_2}} \sinh\left(\frac{x}{\beta_2}\right)^{\gamma_2}\right]$$

[d] Force of mortality function

$$\mu(x) = -\frac{S'(x)}{S(x)} = \frac{\frac{\alpha_1\gamma_1}{\beta_1}\left(\frac{x}{\beta_1}\right)^{\gamma_1-1} e^{-\exp(\frac{x}{\beta_1})^{\gamma_1}} e^{(\frac{x}{\beta_1})^{\gamma_1}} + \frac{\alpha_2\gamma_2}{\beta_2}\left(\frac{x}{\beta_2}\right)^{\gamma_2-1} e^{-\cosh(\frac{x}{\beta_2})^{\gamma_2}} \sinh\left(\frac{x}{\beta_2}\right)^{\gamma_2}}{\alpha_1 e^{-\exp(\frac{x}{\beta_1})^{\gamma_1}} + \alpha_2 e^{-\cosh(\frac{x}{\beta_2})^{\gamma_2}}} \qquad (4)$$

3. Estimation results and models of parameters

Parameters of the proposed CH function in equation (3) can be estimated by the method of non-linear least squares estimation using the sample data of annual life tables. That is:

$$\textit{Minimize} \sum_{all\ x} (S(x) - \hat{S}(x))^2$$

where $S(x)$ is the survival probability at age x available from the life tables and $\hat{S}(x)$ is the corresponding estimate of $S(x)$. Through our numerical computations, we note that different optimization routines may return different estimates of the same parameter, implying that there may be multiple local minima. In order to help eliminate such local minima, we took the liberty of imposing the constraint that α_1 is less than α_2. For empirical illustration, we fit the CH function to annual life tables for the US population by gender from 1950 to 2010 respectively. The estimation is performed on the MATLAB platform and computer programs are available from the authors upon request.

Estimation results of the CH survival function for each year of the sample period from 1950 to 2010 are tabulated in Table 1 for females and in Table 2 for males. As can be observed, the proportion of the variance explained by the CH model is at least 0.9999 for both females and males, with an average of 0.99996 for females and 0.99997 for males. The residual sum squares *(RSS)* are quite low, ranging from 0.00023 to 0.00144 for females and from 0.00018 to 0.00112 for males. Moreover, the mean absolute percentage error (MAPE in %) is rather small for both gender, ranging from 0.05146 to 0.09829 for females and from 0.06398 to 1.32674 for males. The high R^2, low *RSS* and small MAPE indicate that the proposed CH function fits well for the US population when the men and women are evaluated separately.

Figures 2A and 2B plot the observed and estimated values of survival probability for the US females and males in year 2000 by the proposed CH function, the Gompertz-Makehem function and the logistic function of Bongaarts, respectively. As can be observed, the CH function is closer to the actual distribution for the support ranging from adulthood to the oldest-old ages, indicating that it is a better approximation than the other two competing counterparts. The zoomed portion of these functions for ages 65 to 105, which provides a better glimpse of the differences between observed and fitted values along the right-tail, are plotted in Figures 3A and 3B. Both figures reveal small under-estimations of survival probability between ages 75 and 90, and relatively moderate over-estimations in the oldest-old region by the logistic function of Bongaarts and Gompertz-Makeham model. Within this range of ages, the CH function seems to fit best with the observed distribution.

Time series models of parameters

In what follows, we report our experiments with modelling parameters of the CH function using time series models. There are 61 annual observations for each parameter, which are estimated from the US life tables for females and males respectively between the years of 1950 and 2010 inclusive. Panels A and B of Table 3 tabulate the summary statistics for each of the six series of paramters of the CH function. It can be seen that the standard deviations of all series for females are lower than those of males. For both the males and females, the sample kutosis of all series are negative with moderate magnitudes between -1.32 and -0.29. The signs of the sample skewness are mixed with magnitudes in the vacinity of 0, ranging from -0.63 to 0.68.

Figures 4A and 4B present the plots of these six series of parameters, including $\alpha_1(t)$, $\beta_1(t)$, $\gamma_1(t)$, $\alpha_2(t)$, $\beta_2(t)$ and $\gamma_2(t)$. The solid lines denote the estimates of parameters. We can see from these figures that the $\alpha_1(t)$ and $\alpha_2(t)$ series appear to have a flat slope over time. For example, the range of $\alpha_1(t)$ is (0.1439, 0.2487) with a small standard deviation of 0.0241 for females; and is (0.1513, 0.4754) with a standard deviation of 0.0941 for males; whereas the range of $\alpha_2(t)$ is (2.4484, 2.5108) with a small standard deviation of 0.0137 for females and the range is (2.2222, 2.5292) with a standard deviation of 0.0871 for males. This suggests that the scale parameters in the CH function consistently carry weights of relatively constant magnitudes through time. Similarly, the $\beta_1(t)$ series is also relatively constant during the sample period, implying that the youth-to-adulthood component does not have drastic changes over the past few decades. In contrast, the displayed upward drift in the $\beta_2(t)$ series indicates that the old-to-oldest-old component of the CH function has scaled horizontally, thereby suggesting that life expectancies at old ages may have improved. Also, the $\gamma_1(t)$ and $\gamma_2(t)$ series both display upward trends, thereby implying relatively moderate changes in mortality patterns over time. This is consistent with the observed phenomenon that mortality decline accelerates at old ages in developed countries. See Horiuchi and Wilmoth (1998) and Li et al. (2013). We shall provide further discussion in Section 5.

Before modeling the time series of parameters, we perform the augmented Dicky-Fuller (ADF) tests and Phillips-Perron (PP) tests for stationarity. Though not reported here, the test statistics are all insignificant at the 5% level, thereby providing justification for taking first-differencing of the series with a view to make them stationary. After the first-differencing, both the ADF and PP test statistics are found significant at the 5% level for all series under study, indicating rejection of the null hypothesis for the presence of the unit root. Hence, for practical purposes, we shall replace each series by the corresponding rate of change instead of the level change, which is defined as

$$d(c_t) = \frac{c_t - c_{t-1}}{c_{t-1}}$$

where c_t denotes one of the parameter series (α_1, β_1, γ_1, α_2, β_2, γ_2) at time t and $d(c_t)$ denotes its rate of change from integer age (t - 1) to age t. Univariate time series models for each of the parameters expressed in rate of change are identified and estimated under the Bayesian Information Criterion. We find that either AR(0) or AR(1) models are adequate for modeling the parameters in the CH function. Figures 4A and 4B display plots of the predicted values of estimates of parameters which are indicated by dotted lines. They are overlaid on the actual values which are estimates of parameters obtained from the non-linear estimation. As can be observed, the fits are reasonably close across all ages. This is further corroborated by diagnostic checks when the Ljung-Box portmanteau statistics indicate no significant residual autocorrelation. Complete results are available from the authors upon request.

4. Forecasting performance of models

In this section we evaluate the performance of the proposed CH function with respect to forecasts of life expectancies at various ages. Our model forecasts are compared with the actual life expectancies obtained from the *Human Mortality Database*; and also with those forecasts of life expectancies by the Lee-Carter approach and by the shifting mortality model of Bongaarts. Computational details of these two approaches are relegated to Appendices A and B.

Sample data of the US life tables from 1950 to 2005 was fed into the respective models and forecasts of life expectancies until 2050 were obtained. The within-sample forecasts of life expectancies are those covering the period 1950 to 2005. The holdout-sample forecasts are those from 2006-2010 and the out-of-sample forecasts are those beyond 2010. We found that re-estimation of the time series models for parameters of the CH function over the within-sample period of 56 years yielded only small differences from results of those fitted time series models as reported in Tables 1 and 2, where the entire sample data from 1950 to 2010 was used. For out-of-sample forecasting, the following steps were taken to compute survival probabilities at integral ages.

1. Standing at year 2010, forecast the parametric values of the CH function for year 2011;
2. Plug in forecasts of parameters into the CH function as specified in equation (3); and
3. Generate the survival distribution over different ages;
4. Compute the life expectancies at birth and at ages 20, 40, 65 and 80; and
5. Repeat Steps 1-4 for another calendar year.

Tables 4A and 4B display the actual life expectancies at various ages for the holdout-sample period alongside their forecasts by the CH model, Lee-Carter model and the Bongaarts model. As can be observed, the CH model yields reasonably accurate forecasts compared to the actual ones as well as those by the other two models. In addition, Figure 5 provides plots of forecasts of life expectancies at various ages for females during the period 1950-2050 under the CH model, the Lee-Carter model and the model by Bongaarts, respectively. Similar plots of forecasts of life expectancies for males are provided in Figure 6. The solid line in these plots indicates the actual life expectancies computed from the life tables. As can be gleaned from these figures, forecasts of life expectancies by the proposed CH model for females and males match the empirical life expectancies reasonably well at ages 0, 20 40, 65 and 80 respectively in both the within-sample and holdout sample periods. In comparison, forecasts of life expectancies at birth by the CH model and the Lee-Carter model are reasonably close to the actual life expectancies throughout the entire sample period, with less satisfactory performance by those forecasts by Bongaarts. For forecasts of life expectancies at ages 20, 40 and 65, the CH model remains remarkably close to the actual ones and outperforms the other two models. For forecasts of life expectancy at age 80, both the CH model and the Bongaarts' model outperform the Lee-Carter model which produces upward-biased forecasts.

Forecasting performance of the CH model, the Lee-Carter model and the model by Bongaarts can be further assessed quantitatively by employing two commonly used measures. They are the mean absolute error (MAE) and the mean absolute percentage error (MAPE). The MAEs and MAPEs for forecasting life expectancies of these three models are reported in Table 5. We first concentrate on the CH model. As can be observed from the table, the errors are relatively small by the MAE and MAPE criterion. For females, the computed MAE and MAPE statistics for the in-sample period grow from (0.159 years, 0.209%) at birth to (0.210 years, 1.206%) at age 65, and decrease to (0.107 years, 1.274%) at age 80. As regards the MAE and MAPE statistics computed through the holdout sample period, the errors grow from (0.064 years, 0.079%) at age 0 to (0.348 years, 1.706%) at age 65; and decrease to (0.125 years, 1.259%) at age 80. Similar patterns are observed for males, with the MAPE statistics being always greater than those for females.

When comparing forecasts of life-expectancies at birth and at 20, both the Lee-Carter model and the proposed CH model produce reasonably good fit over the within-sample and holdout-sample periods. However, the proposed CH model outperforms the Lee-Carter model in the withhold sample period. While the differences in MAPE for these two models are small, the forecasting errors compound over time, thereby leading to vastly different predictions over longer duration. For example, the proposed CH model predicts a life expectancy at birth of 88.5 years by 2050, as compared to the Lee-Carter prediction of 84.9 years. We note in passing that errors in forecasts of life expectancies at birth and at 20 by the Bongaarts method are not strictly comparable as this approach is mainly designed for modelling mortality rates above age 25. Though not reported here, the Lee-Carter model provides less satisfactory forecasts of life expectancies at ages above 50. This may be due to the estimation procedures which involve matching life expectancies in the re-estimation of $k(t)$. As the mortality rates at younger ages affects the life expectancy at birth more greatly, the Lee-Carter model places higher weights on them and discounts the effect of mortality rates at older ages. Thus, its prediction of mortality at higher ages may be less than satisfactory.

For forecasts of life expectancies at ages 40 and 65, both the MAE and MAPE statistics are reasonably small. The error statistics for the CH model are even smaller than those produced by Bongaarts in both within-sample and withhold-sample periods. In contrast, forecasts of life expectancy at 80 by model of Bongaarts in the withhold-sample period perform better than those from the CH function. Upon a closer inspection, we find that the CH method consistently underestimates the life expectancy. Such an underestimation could be easily corrected by subtracting the mean error from the within-sample estimates to adjust the level. With this correction, the forecasts made by the CH function are comparable to those out-of-sample forecasts by Bongaarts. In summary, forecasts of life expectancies at various ages by the proposed CH model are at least comparable, if not better, to those from the Bongaarts and Lee-Carter.

5. Some discussions

In what follows, we report simulation results designed for approximating the 95% confidence interval for forecasts of life expectancies at various ages by the CH model, and discuss some of its implications.

For simplicity, the errors of each fitted univariate time series of parameters of the CH function are assumed to be independently normally distributed with variances obtained from AR(1) estimations. For a given calendar year beyond 2005, simulated values of parameters are generated from the fitted time series models, which are then fed into equation (3) to obtain the survival distribution. The life expectancies at ages 0, 20, 40, 65 and 80 are computed from the simulated survival distribution. Repeating the same procedure 100,000 times, we are able to map out a distribution of life expectancies at various ages, and from which the required confidence intervals for each of the life expectancies can be obtained accordingly. It took us about 7.3 hours for 100,000 iterations when the MATLAB programs are executed on a 1.6GHz Xeon processor without parallel processing. The same procedures implemented on a GPGPU (an overclocked Nvidia GeForce GTX 960) using CUDA 7.0 (C++ API) took less than 5 seconds to complete the 100,000 iterations, demonstrating that with careful design and proper hardware, the model is suitable for 'online' analysis.

Panels A and B of Table 6 report the simulated 95% confidence intervals of forecasts of life expectancies for females and males by the CH model from 2006 to 2050. For easy visualization, Figures 7 and 8 plot the corresponding confidence bounds for such forecasts covering the same period. As can be observed, for all mean forecasts of life expectancies at ages 0, 20, 40, 65 and 80 in the holdout-sample period, the actual values reported in Tables 4A and 4B are within the 95% confidence intervals obtained by simulation.

Plots A and B of Figure 9 display the forecasted decennial survival distributions of the US females and males by the CH model from 2010 to 2050. As can be seen from these plots, the distribution for year 2010 lies right at the bottom, with the other four distributions gradually shifting up to the right side over the period of 40 years. Particularly, there is a decrease in the distance between decennial survival distributions for ages below 70, and a slight increase in the spacing for ages above 70. This implies a *deceleration* in youth and adult mortality decline, and an *acceleration* of the old to oldest-old mortality decline. The corresponding plots of forecasted mortality by the CH model are displayed in Figure 10. Our findings are consistent with those by Horiuchi and Wilmoth (1998) and Li et al. (2013).

Intuitively, the upward shift in the decennial distributions implies improvement in survival probability at all ages. This is corroborated by the increase in life expectancies at various ages. As can be gleaned from Column 2 in Panels A and B of Table 6, forecasts of the life expectancy at birth in the US for males by the CH model will rise to 82.01 by 2025 and to 95.47 by 2050, with the corresponding 95% confidence intervals of (71.70, 95.89) and (73.94, 132.85) respectively. As for females, forecasts of the life expectancy at birth by the CH model will rise to 84.33 years by 2025 and to 90.29 years by 2050, with the corresponding 95% confidence intervals of (81.26, 87.44) and (84.85, 95.89), respectively.

Our findings indicate that the proposed CH model is able to capture the stylized feature of 'rotation' of age-pattern of mortality decline through changes in the shape parameters. See also Plots A and B of Figure 11, which indicate the relative change in mortality rates at different ages by females and males respectively. However, the forecasted survivorship schedule for 2050 may indicate the danger of extrapolating current trends and expecting them to remain the same over the next few decades. The seemingly unnatural bend, occurring between ages of 70 and 80, in the curve is a result of the mortality rates at those ages converging at the same rate as adult mortality rates, before rising again. At the risk of oversimplification, it is a plausible but counter-intuitive scenario. Most probably, the observed 'rotation' is merely a temporary phenomenon and that the trend may reverse itself in the future to smooth out the rate of change of mortality decline.

6. Concluding remarks

In this paper we have introduced a new survival function to model adult mortality and to forecast life expectancies at various ages by gender. The proposed model comprises the youth-to-adulthood component and the old-to-oldest-old component. In combination, these two components are shown to be able to closely fit adult mortality of the US men and women from the life tables for each year covering the period from 1950 to 2010.

We first obtain estimates of parameters of the proposed survival function for each year over the sample period 1950-2010 of the US data. Such estimates of each parameter are treated as a sample of time series for that particular parameter. Standard techniques are then applied to each time series to obtain lower-order ARMA models. This tractable 2-step approach is able to provide forecasts of life expectancies reasonably close to the actual ones. Moreover, we find evidence that forecasts of life expectancies by the proposed model compare favorably with those obtained by the popular Lee-Carter model (1992) and with those forecasts by the shifting logistic model proposed by Bongaarts (2005). At the risk of oversimplification, the proposed model seems to be able to capture an interesting feature about rotation of age-pattern of mortality decline through changes in the shape parameters.

Conceivably it is not our intention to claim that the proposed survival function represents the most adequate model for the US population. Definitely other mortality/survival models could also provide reasonably accurate forecasts of life expectancies at various ages. To our knowledge, the proposed survival function, as an alternative, is able to cover both the youth-to-adulthood component and the old-to-oldest-old component of the survival distribution that are not explicitly captured in other studies. One possible caveat in our study is that we do not engage multiple time series models to fix the possible correlations among estimates of the parameters of the CH function. This will be taken up in the future.

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Appendix A – The Lee-Carter Model

The Lee-Carter model (1992) is specified as follows:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$$

where the matrix of the logarithm of mortality rates over time $\ln(m_{x,t})$ is decomposed into a constant vector ($\mathbf{a}_x$) dependent on age x and the first principle component, $b_x k_t$, with b_x being the rate of decline in mortality at each age x and k_t is a time-varying index of level of morality. Girosi and King (2007) give an excellent discussion of the model.

We obtain a_x by finding the average of each age-specific mortality rate over time. b_x is obtained by extracting the first left-singular vector of $\ln(m_{x,t})$ - a_x. In addition, k_t is obtained directly by the method of singular value decomposition, with re-estimation by matching life expectancies at birth.

Since a_x and b_x are assumed to be static, the mortality rates and hence, the life-expectancy at age x at future can be obtained by just forecasting k_t. We follow the convention to model and forecast k_t using standard ARMA time series models.

Given its simplicity and ease of implementation, the Lee-Carter approach has been applied to investigate mortality rates in many countries. However, the assumption that b_x is constant has been shown to fail in some countries in recent decades, leading Li et al. (2013) to extend the original Lee-Carter model to factor in the 'rotation' of age patterns of mortality decline. In our study, the extended Lee-Carter method is not used since its approach to forecasting life expectancies is similar to the original Lee-Carter method proposed in 1992.

Appendix B – Shifting logistic model of Bongaarts

Bongaarts' shifting logistic model:

$$\mu(x,t) = \alpha(t) + \frac{\beta e^{\gamma(t)x}}{1 + \beta e^{\gamma(t)x}}$$

The projection procedure consists of four steps:

1) Estimate the parameters on right side of the equation to mortality schedules over a period and take the average of the estimated β.
2) Re-estimate $\alpha(t)$ and $\gamma(t)$ in equation (6) by fixing β at the average value obtained in Step 1.
3) Extrapolate the $\alpha(t)$ and $\gamma(t)$ obtained using ARIMA time-series.
4) Reconstruct the future mortality rates by plugging forecasted $\alpha(t)$ and $\gamma(t)$ into the equation.

Based on the forecasted mortality rate, we can easily compute the life-expectancy at any age. However, Bongaarts cautions that the shifting logistic model does not apply to mortality patterns at ages younger than 25, making any life-expectancy forecasts below the age of 25 unreliable and prone to errors.

Figure 2A: Fitted survival probabilities of the US females for year 2000

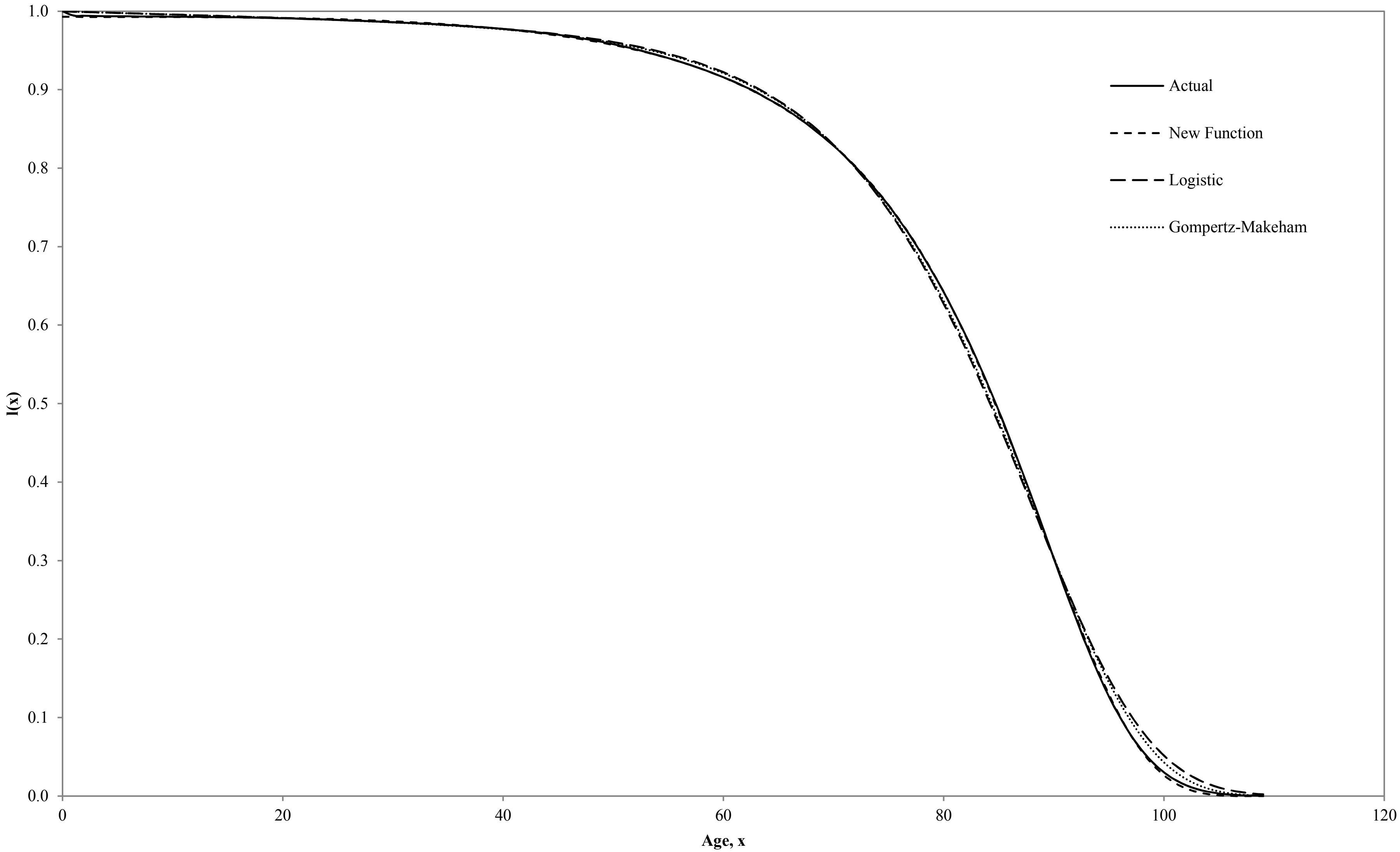


Figure 2B: Fitted survival probabilities of the US males for year 2000

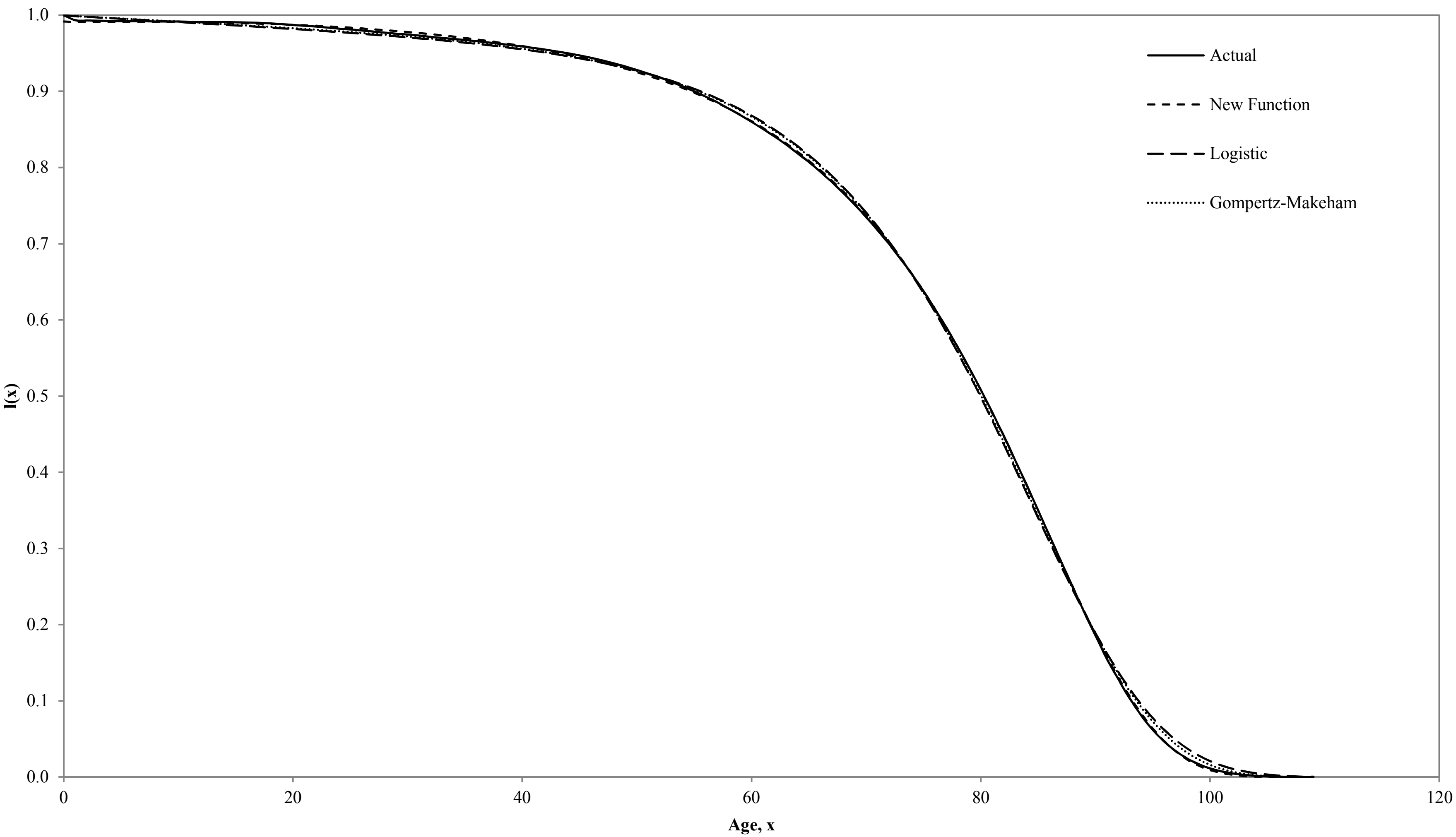


Figure 3A: Zoomed portion of the fitted survival probabilities of the US females for year 2000

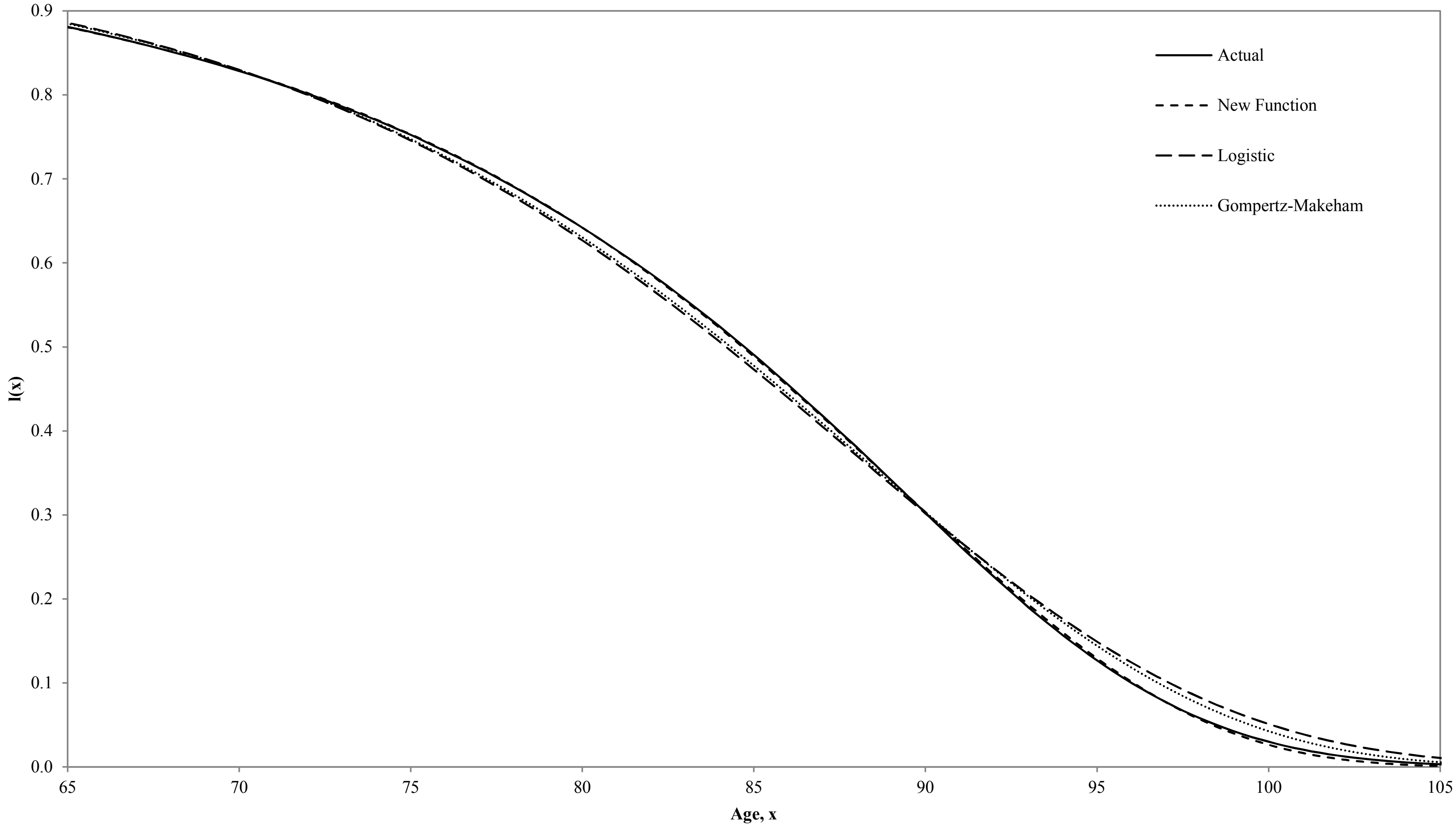


Figure 3B: Zoomed portion of the fitted survival probabilities of the US males for year 2000

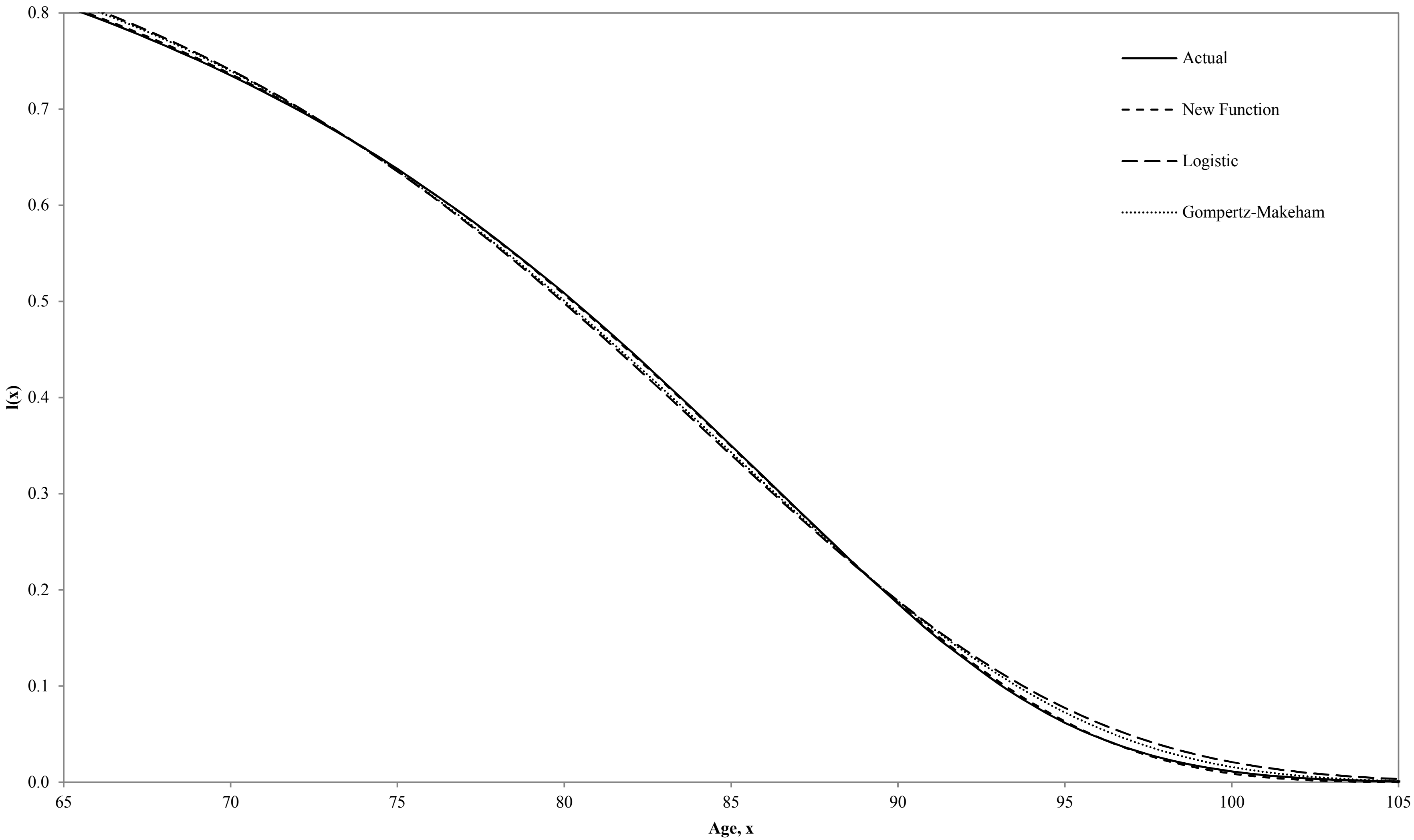
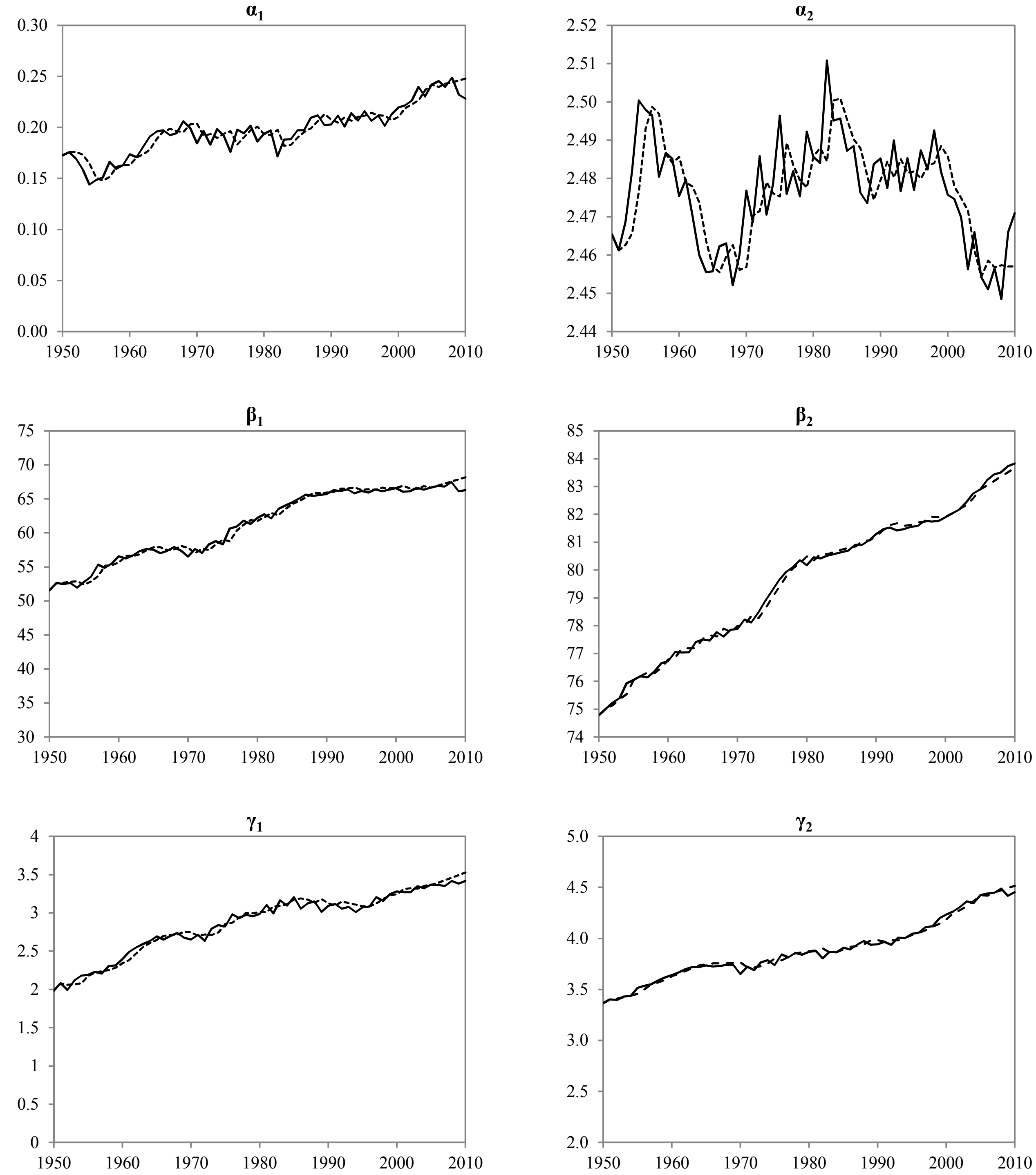
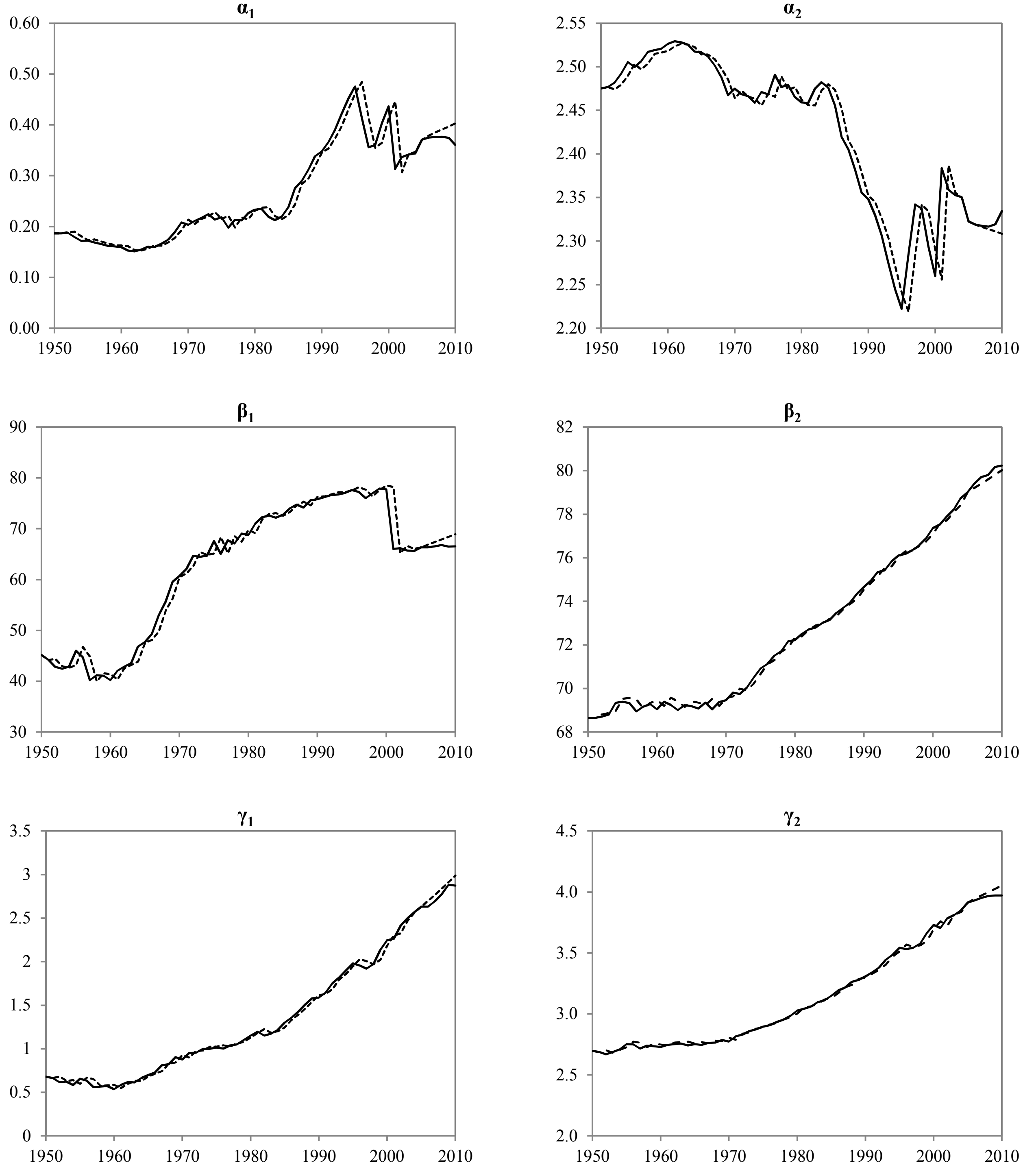


Figure 4A: Estimates of CH parameters for the US females from 1950 to 2010



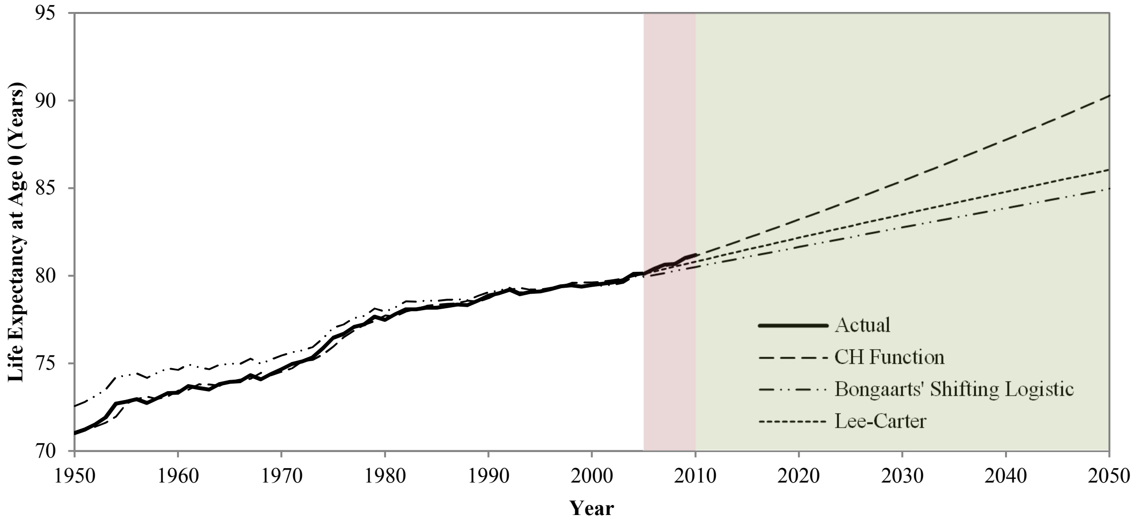
Note: Solid lines denote actual estimated coefficients. Dotted lines show the predicted values.

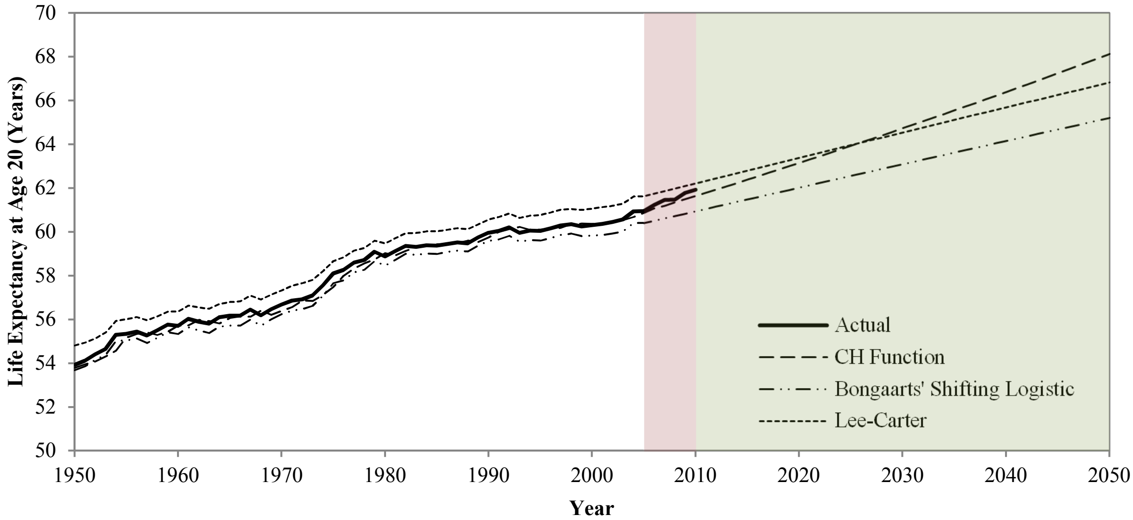
Figure 4B: Estimates of CH parameters for the US males from 1950 to 2010

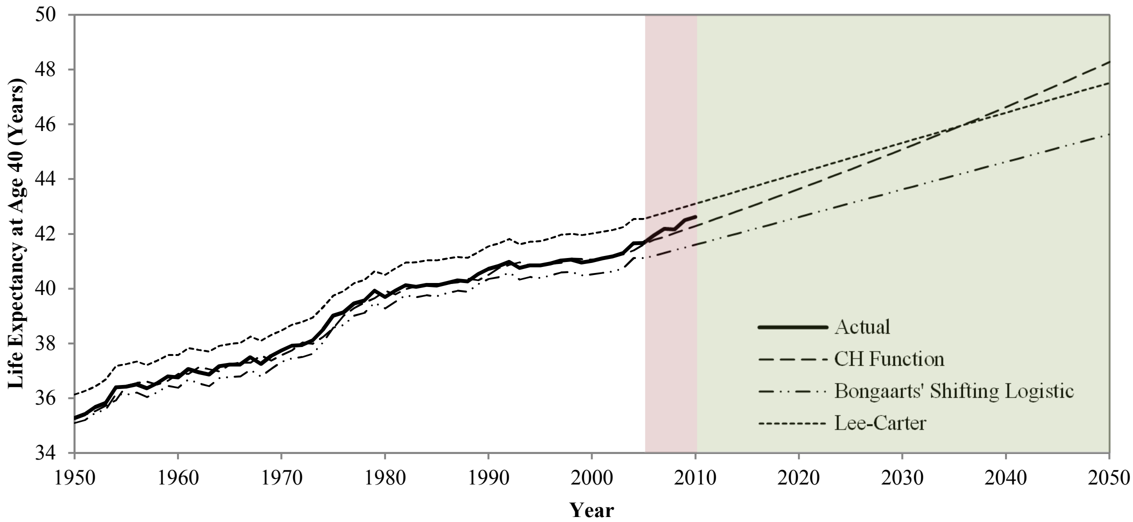


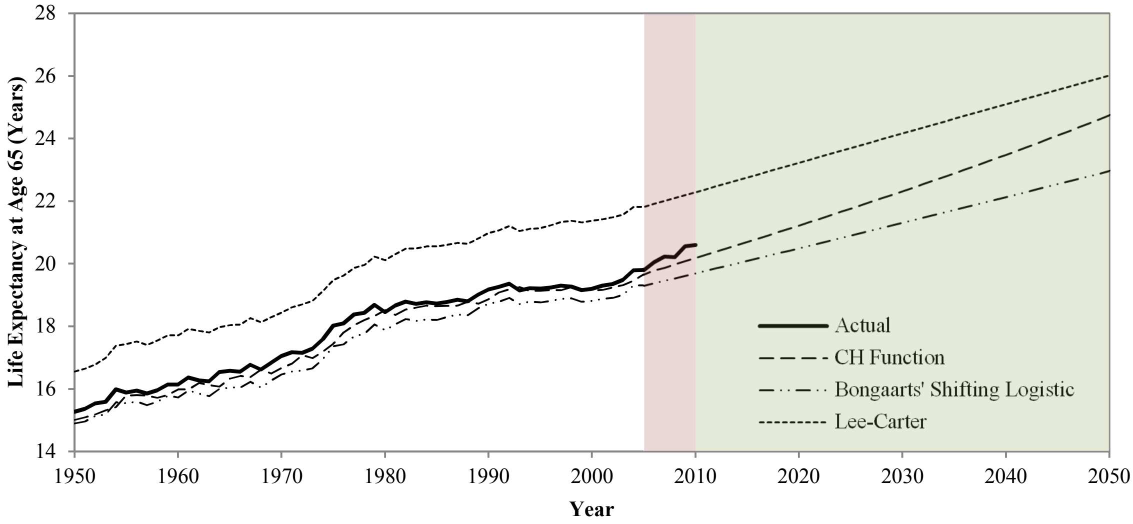
Note: Solid lines denote actual estimated coefficients. Dotted lines show the predicted values.

Figure 5: Forecasts of life expectancies at ages 0, 20, 40, 65 and 80 for the US females by different models

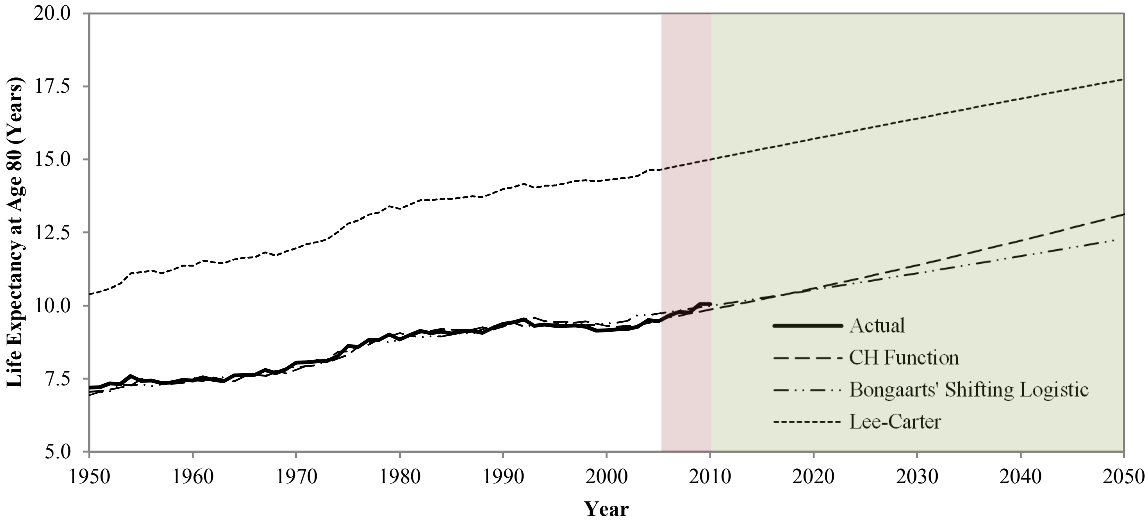






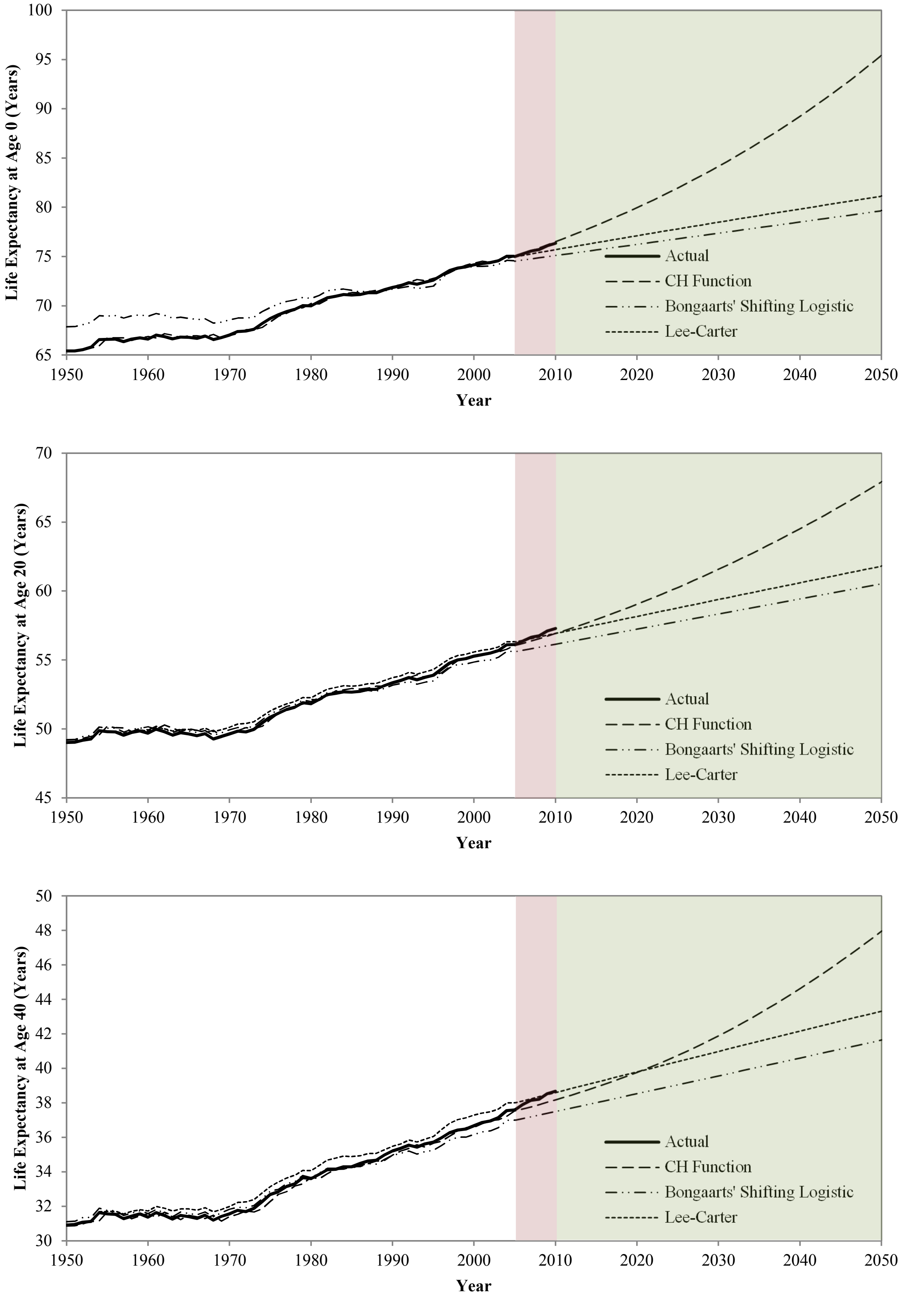


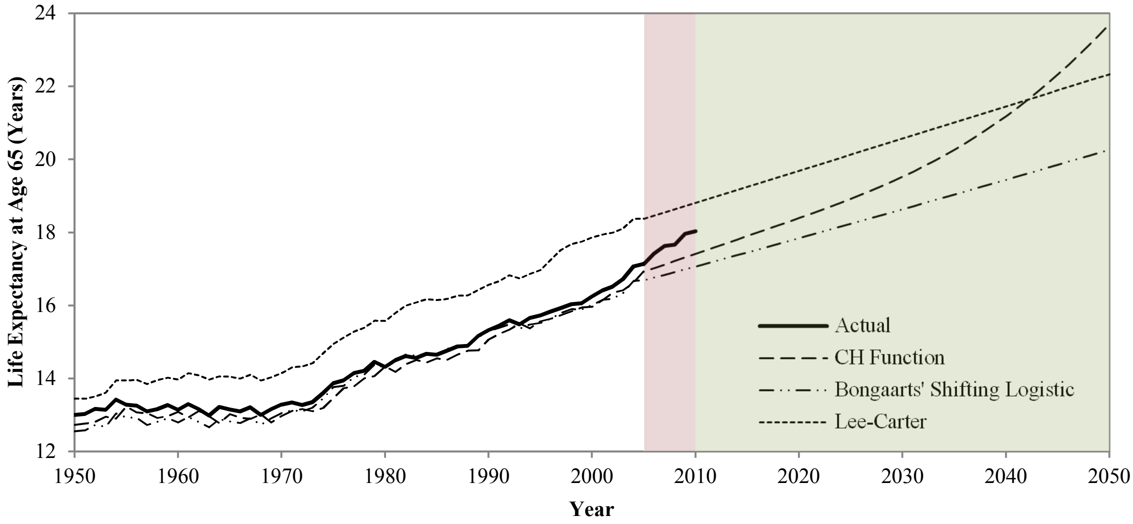
Life Expectancy at Age 65 (Years)
Year
Actual
CH Function
Bongaarts' Shifting Logistic
Lee-Carter



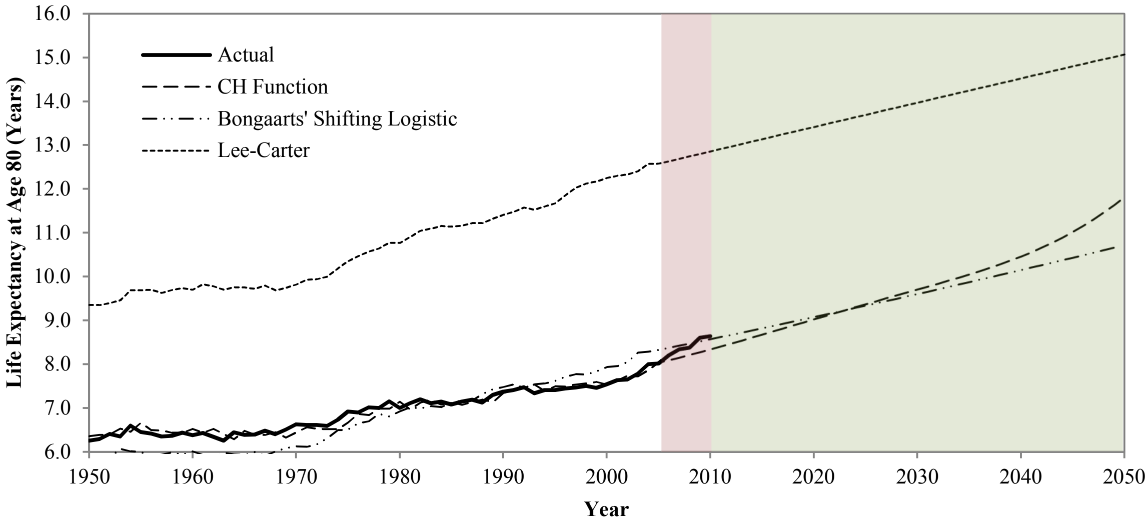
Life Expectancy at Age 80 (Years)
Year
Actual
CH Function
Bongaarts' Shifting Logistic
Lee-Carter

Figure 6: Forecasts of life expectancies at ages 0, 20, 40, 65 and 80 for the US males by different models





24
22
20
18
16
14
12
Life Expectancy at Age 65 (Years)
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
Year
Actual
CH Function
Bongaarts' Shifting Logistic
Lee-Carter



16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
Life Expectancy at Age 80 (Years)
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
Year
Actual
CH Function
Bongaarts' Shifting Logistic
Lee-Carter

Figure 7: 95% confidence bounds for forecasts of life expectancy by the CH model for the US females

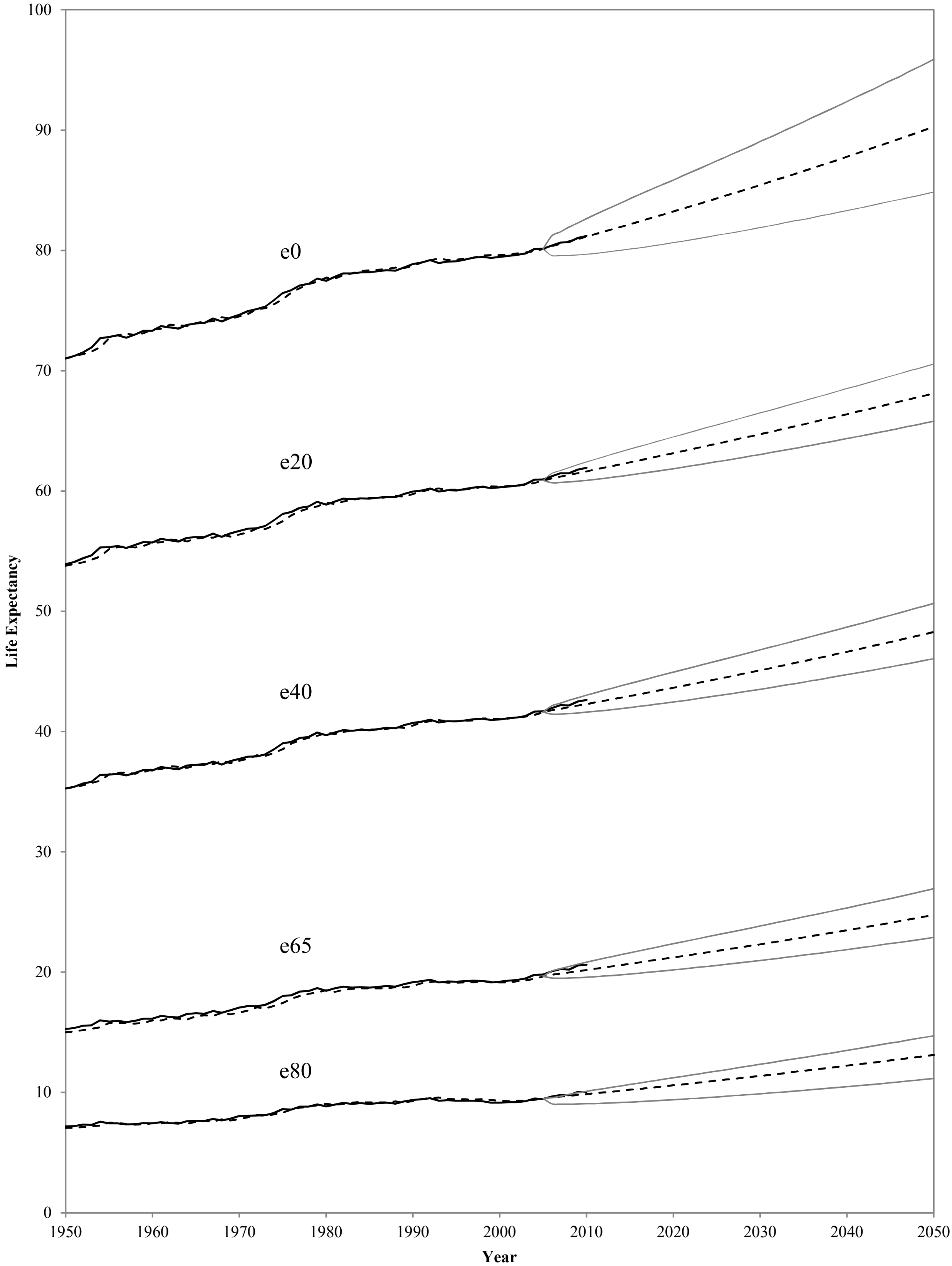


Figure 8: 95% confidence bounds for forecasts of life expectancy by the CH model for the US males

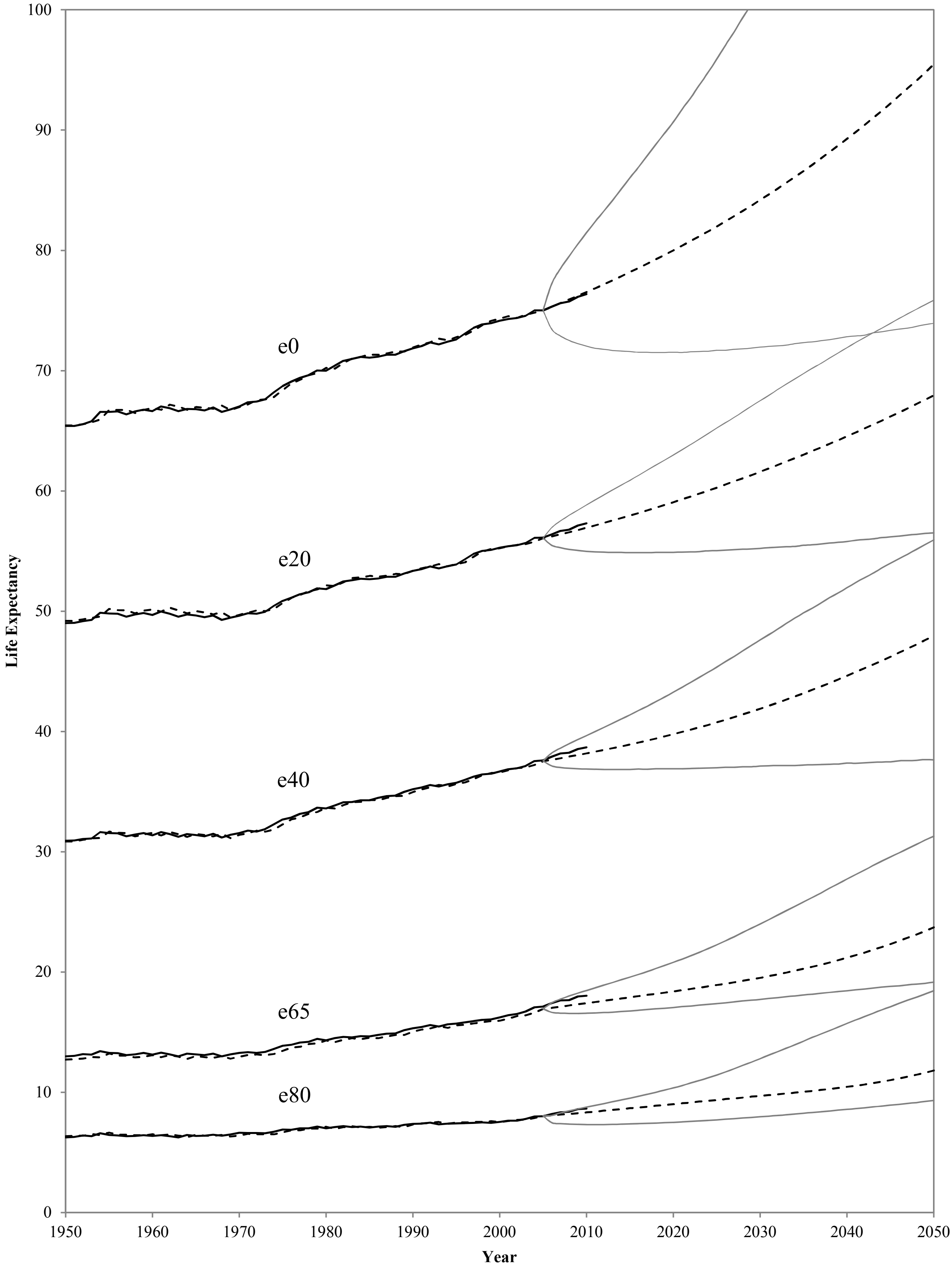
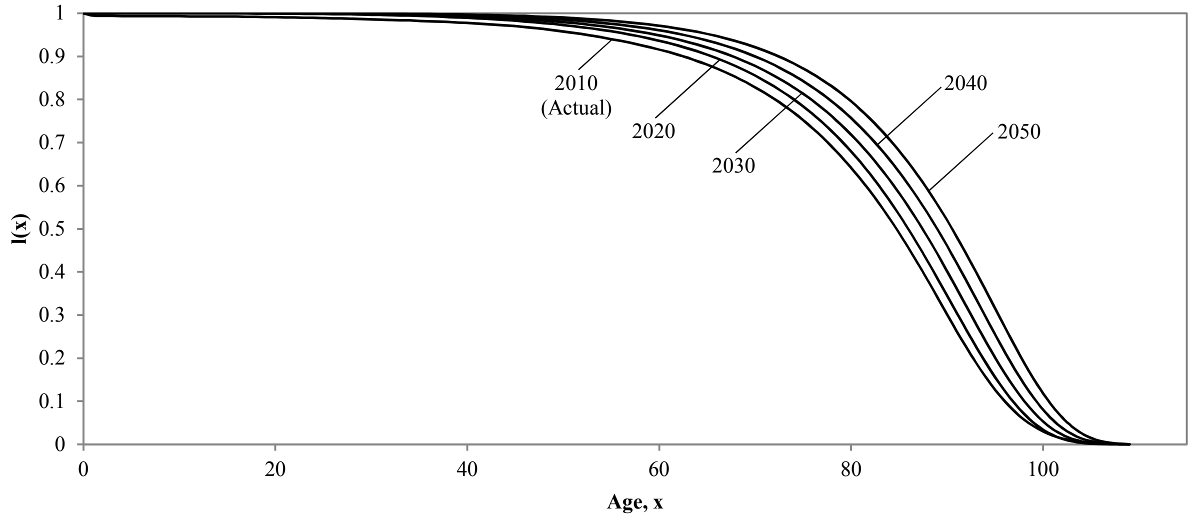


Figure 9: Forecasted survival distributions

Plot A: US females



Plot B: US males

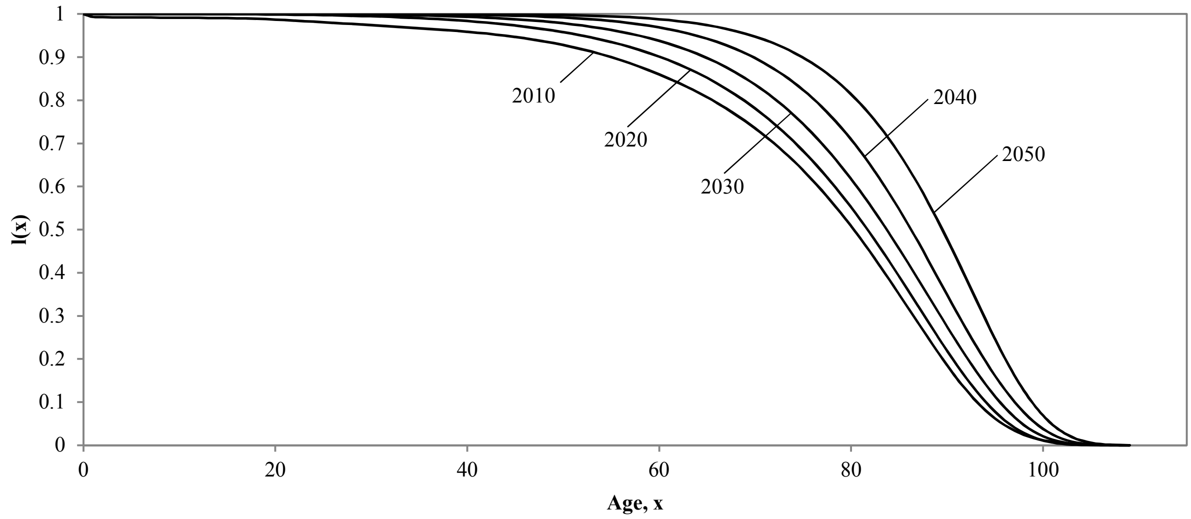
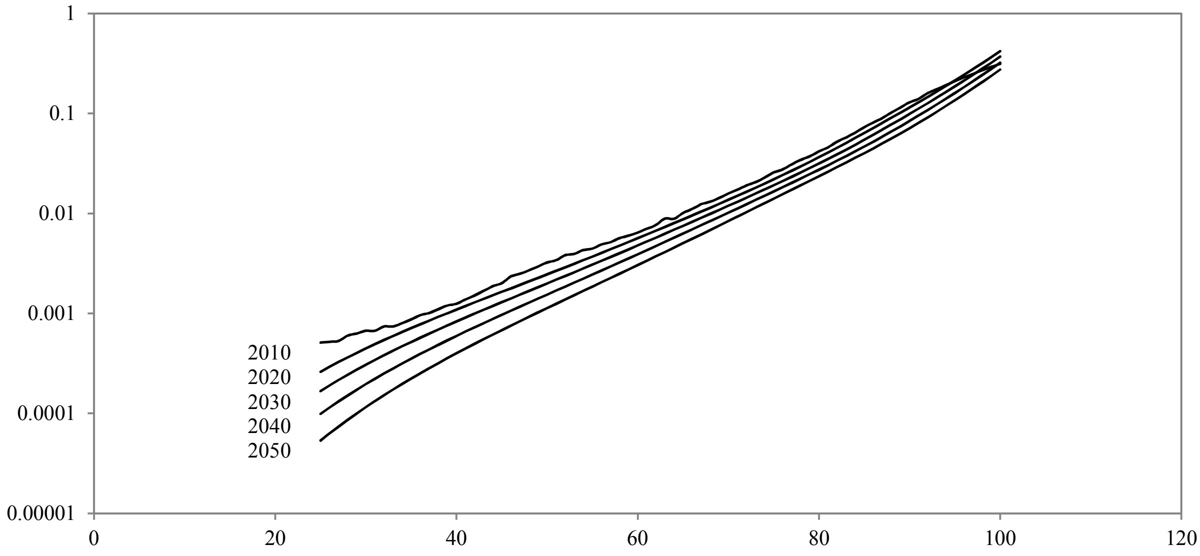


Figure 10: Forecasted Mortality

Plot A: US females



Plot B: US males

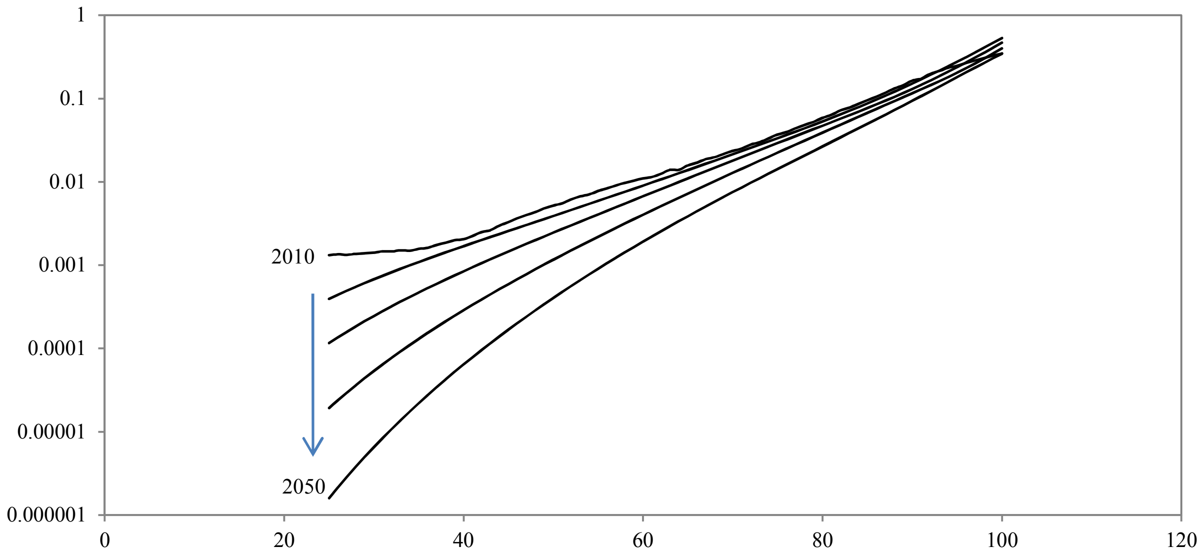
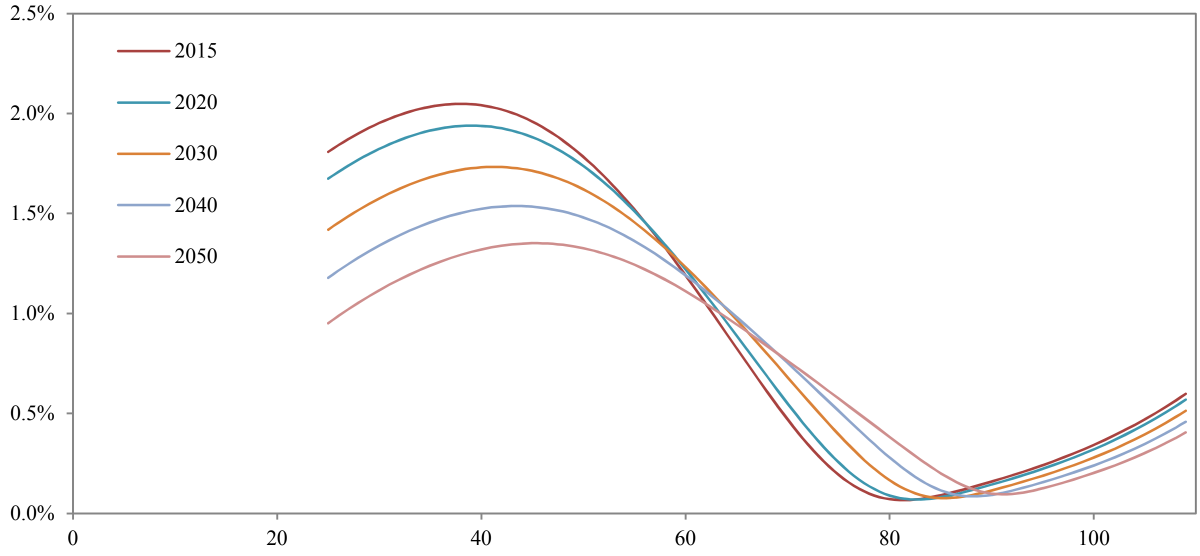


Figure 11: Forecasted Relative Change in Mortality Rates

Plot A: US females



Plot B: US males

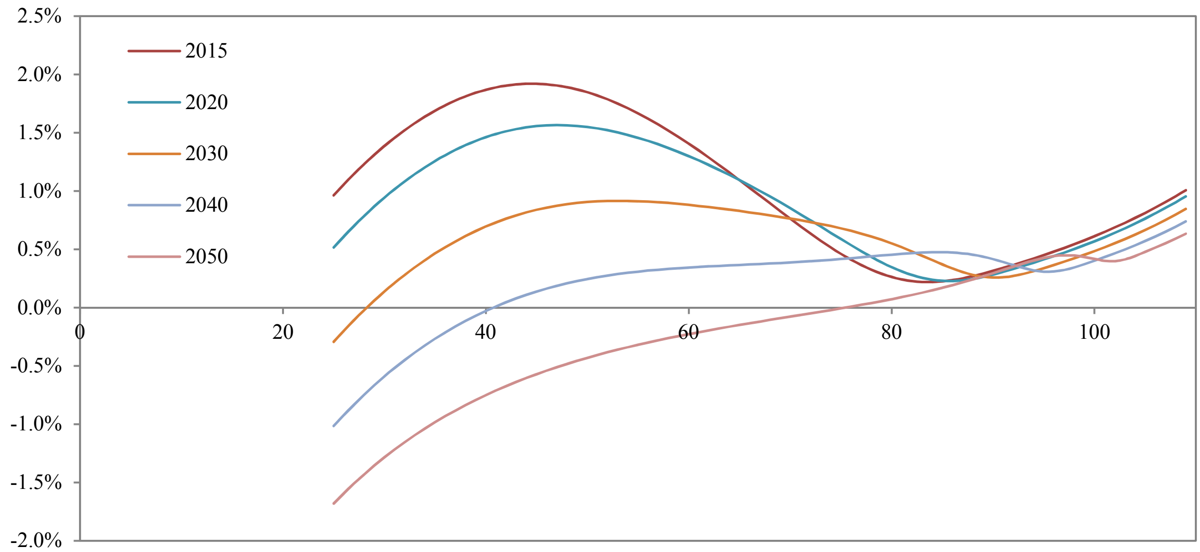


Table 1: Estimation results of the CH function fitted to the US females

Year	α_1	β_1	γ_1	α_2	β_2	γ_2	RSS	MAPE	R^2
1950	0.1727	51.5717	1.9846	2.4654	74.7828	3.3658	0.00136	0.09723	0.99991
1951	0.1756	52.6495	2.0821	2.4612	75.0172	3.4029	0.00144	0.09829	0.99990
1952	0.1692	52.4760	1.9937	2.4686	75.2309	3.3980	0.00139	0.09407	0.99991
1953	0.1591	52.6457	2.1204	2.4818	75.4055	3.4309	0.00129	0.09530	0.99991
1954	0.1439	51.9303	2.1797	2.5004	75.9274	3.4346	0.00118	0.08867	0.99992
1955	0.1484	52.7936	2.1923	2.4979	76.0501	3.5152	0.00115	0.09469	0.99992
1956	0.1510	53.5746	2.2299	2.4965	76.1791	3.5343	0.00112	0.09518	0.99993
1957	0.1663	55.3140	2.2066	2.4804	76.1394	3.5505	0.00105	0.09437	0.99993
1958	0.1593	54.8868	2.3079	2.4866	76.3507	3.5872	0.00109	0.09524	0.99993
1959	0.1629	55.5321	2.3134	2.4852	76.6543	3.6183	0.00105	0.09377	0.99993
1960	0.1736	56.5707	2.3940	2.4754	76.7220	3.6400	0.00098	0.09297	0.99993
1965	0.1973	57.5335	2.6937	2.4557	77.5059	3.7357	0.00100	0.09263	0.99993
1970	0.1845	56.5123	2.6508	2.4768	77.8822	3.6507	0.00076	0.07986	0.99995
1980	0.1937	62.1946	2.9868	2.4856	80.1749	3.8695	0.00056	0.06650	0.99996
1985	0.1972	64.4603	3.2035	2.4872	80.6421	3.9102	0.00048	0.06196	0.99997
1990	0.2028	65.6888	3.0926	2.4852	81.2919	3.9433	0.00041	0.05486	0.99997
2000	0.2196	66.6032	3.2805	2.4757	81.8957	4.2328	0.00029	0.06432	0.99998
2001	0.2215	66.0330	3.2709	2.4746	82.0329	4.2620	0.00028	0.06460	0.99998
2002	0.2263	66.1424	3.2699	2.4699	82.1757	4.3067	0.00026	0.06505	0.99998
2003	0.2399	66.6277	3.3483	2.4563	82.4387	4.3622	0.00027	0.06478	0.99998
2004	0.2302	66.3398	3.3216	2.4660	82.7477	4.3482	0.00027	0.06131	0.99998
2005	0.2420	66.6771	3.3696	2.4543	82.8955	4.4250	0.00026	0.06325	0.99998
2006	0.2453	66.8828	3.3670	2.4511	83.2231	4.4405	0.00026	0.06047	0.99998
2007	0.2398	66.8101	3.3535	2.4566	83.4306	4.4468	0.00025	0.05822	0.99998
2008	0.2487	67.4612	3.4152	2.4484	83.5032	4.4879	0.00023	0.05881	0.99998
2009	0.2322	66.1193	3.3830	2.4661	83.7302	4.4166	0.00023	0.05354	0.99998
2010	0.2282	66.2780	3.4154	2.4709	83.8294	4.4516	0.00023	0.05405	0.99998

Table 2: Estimation results of the CH function fitted to the US males

Year	α_1	β_1	γ_1	α_2	β_2	γ_2	RSS	MAPE	R^2
1950	0.1863	45.1764	0.6798	2.4753	68.6514	2.6966	0.00112	0.80658	0.99993
1951	0.1863	44.2271	0.6644	2.4769	68.6482	2.6880	0.00112	0.81064	0.99993
1952	0.1874	42.8369	0.6182	2.4820	68.7047	2.6685	0.00108	0.81359	0.99993
1953	0.1792	42.4397	0.6233	2.4920	68.7994	2.6892	0.00101	0.81234	0.99994
1954	0.1718	42.8739	0.5834	2.5054	69.3359	2.7092	0.00082	0.69648	0.99995
1955	0.1723	46.0134	0.6568	2.4999	69.3903	2.7513	0.00083	0.95362	0.99995
1956	0.1685	44.7311	0.6349	2.5064	69.3336	2.7493	0.00086	0.91179	0.99995
1957	0.1652	40.2201	0.5590	2.5170	68.9482	2.7171	0.00087	1.10851	0.99995
1958	0.1618	41.2040	0.5667	2.5191	69.1670	2.7399	0.00081	1.06757	0.99995
1959	0.1607	41.0989	0.5724	2.5208	69.2827	2.7349	0.00081	0.96434	0.99995
1960	0.1592	40.2075	0.5363	2.5262	69.0460	2.7291	0.00076	1.22872	0.99995
1965	0.1601	47.7375	0.6991	2.5169	69.1886	2.7526	0.00079	0.85867	0.99995
1970	0.2032	60.7007	0.8775	2.4750	69.4634	2.7743	0.00073	1.05775	0.99995
1975	0.2180	67.5644	1.0156	2.4684	70.9158	2.8946	0.00058	0.91184	0.99996
1980	0.2334	68.7373	1.1517	2.4590	72.2218	3.0281	0.00044	0.65568	0.99997
1985	0.2383	72.7361	1.3012	2.4556	73.1393	3.1487	0.00030	0.68777	0.99998
1990	0.3479	75.8258	1.5883	2.3479	74.6621	3.3070	0.00028	0.40947	0.99998
2000	0.4364	77.7818	2.2465	2.2598	77.3655	3.7293	0.00020	0.08493	0.99999
2001	0.3128	66.0294	2.2648	2.3839	77.5270	3.7049	0.00020	0.07618	0.99999
2002	0.3363	66.1673	2.4139	2.3588	77.8924	3.7826	0.00024	0.07610	0.99998
2003	0.3415	65.6977	2.5008	2.3527	78.2099	3.8106	0.00027	0.07493	0.99998
2004	0.3437	65.6332	2.5670	2.3505	78.7311	3.8498	0.00031	0.07200	0.99998
2005	0.3707	66.3566	2.6304	2.3225	79.0045	3.9129	0.00034	0.07224	0.99998
2006	0.3750	66.3214	2.6319	2.3188	79.3934	3.9308	0.00036	0.06977	0.99998
2007	0.3758	66.5173	2.6929	2.3173	79.6995	3.9512	0.00040	0.06788	0.99997
2008	0.3768	66.7779	2.7760	2.3166	79.7970	3.9671	0.00040	0.06755	0.99997
2009	0.3745	66.5086	2.8822	2.3193	80.1715	3.9702	0.00039	0.06440	0.99997
2010	0.3609	66.5329	2.8747	2.3342	80.2327	3.9713	0.00039	0.06398	0.99997

Table 3: Summary statistics of estimates of parameters

Panel A: US females

	α_1	β_1	γ_1	α_2	β_2	γ_2
Mean	0.1973	61.1532	2.8659	2.4765	79.6051	3.8880
Maximum	0.2487	67.4612	3.4154	2.5108	83.8294	4.4879
Minimum	0.1439	51.5717	1.9846	2.4484	74.7828	3.3658
Standard deviation	0.0241	5.0173	0.3998	0.0137	2.5167	0.2934
Skewness	0.0099	-0.3556	-0.6298	-0.0476	-0.2152	0.4153
Kurtosis	-0.2848	-1.3206	-0.6581	-0.5125	-1.1334	-0.5416

Panel B: US males

	α_1	β_1	γ_1	α_2	β_2	γ_2
Mean	0.2642	62.3172	1.3664	2.4232	72.9080	3.1532
Maximum	0.4754	77.8917	2.8822	2.5292	80.2327	3.9713
Minimum	0.1513	40.2075	0.5363	2.2222	68.6482	2.6685
Standard deviation	0.0941	12.7492	0.7192	0.0871	3.7295	0.4310
Skewness	0.5287	-0.5377	0.6792	-0.5822	0.5061	0.6184
Kurtosis	-1.1224	-1.1919	-0.8207	-1.0312	-1.1102	-1.0450

Table 4A: Forecasts of life expectancies at ages 0, 20, 40 and 65 for the US females by various models

	Actual					CH Function					Bongaarts					Lee-Carter				
Age	**0**	**20**	**40**	**65**	**80**	**0**	**20**	**40**	**65**	**80**	**0**	**20**	**40**	**65**	**80**	**0**	**20**	**40**	**65**	**80**
2006	80.41	61.22	41.95	20.06	9.66	80.38	61.07	41.79	19.80	9.62	80.07	60.51	41.21	19.39	9.73	80.28	61.75	42.67	21.92	14.72
2007	80.65	61.46	42.19	20.24	9.79	80.56	61.20	41.90	19.88	9.66	80.19	60.62	41.32	19.47	9.79	80.42	61.86	42.78	22.01	14.79
2008	80.69	61.47	42.18	20.22	9.77	80.76	61.35	42.04	19.99	9.74	80.30	60.73	41.42	19.55	9.84	80.56	61.98	42.89	22.11	14.86
2009	81.04	61.79	42.51	20.57	10.05	80.96	61.50	42.17	20.09	9.80	80.42	60.84	41.52	19.63	9.89	80.70	62.10	43.00	22.20	14.93
2010	81.21	61.92	42.63	20.61	10.05	81.16	61.64	42.30	20.19	9.87	80.53	60.95	41.62	19.71	9.95	80.84	62.21	43.11	22.30	15.00

Table 4B: Forecasts of life expectancies at ages 0, 20, 40 and 65 for the US males by various models

	Actual					CH Function					Bongaarts					Lee-Carter				
Age	**0**	**20**	**40**	**65**	**80**	**0**	**20**	**40**	**65**	**80**	**0**	**20**	**40**	**65**	**80**	**0**	**20**	**40**	**65**	**80**
2006	75.31	56.40	37.91	17.43	8.21	75.33	56.23	37.66	17.04	8.10	74.69	55.73	37.11	16.77	8.33	75.17	56.46	38.13	18.47	12.63
2007	75.60	56.67	38.16	17.64	8.35	75.63	56.40	37.79	17.13	8.16	74.80	55.84	37.21	16.85	8.38	75.31	56.58	38.25	18.56	12.69
2008	75.74	56.77	38.23	17.67	8.38	75.93	56.58	37.92	17.23	8.22	74.91	55.95	37.32	16.93	8.43	75.45	56.70	38.37	18.64	12.75
2009	76.13	57.12	38.53	17.97	8.61	76.24	56.77	38.05	17.33	8.28	75.03	56.06	37.42	17.00	8.48	75.60	56.83	38.49	18.73	12.80
2010	76.37	57.30	38.69	18.04	8.64	76.55	56.95	38.19	17.43	8.35	75.14	56.17	37.52	17.08	8.53	75.74	56.95	38.62	18.82	12.86

Table 5: Comparison of forecast errors in life expectancies for the US population by various models

Age	Criteria	Female CH	Female Bongaarts	Female LC	Male CH	Male Bongaarts	Male LC
0	*MAE (within)*	0.159	0.664	0.002	0.168	1.164	0.002
	MAE (holdout)	0.064	0.497	0.241	0.103	0.917	0.376
	MAPE (within)	0.209	0.896	0.003	0.243	1.724	0.003
	MAPE (holdout)	0.079	0.614	0.298	0.136	1.208	0.495
20	*MAE (within)*	0.174	0.402	0.670	0.180	0.252	0.328
	MAE (holdout)	0.220	0.845	0.408	0.267	0.904	0.173
	MAPE (within)	0.304	0.692	1.157	0.354	0.485	0.631
	MAPE (holdout)	0.357	1.371	0.663	0.469	1.588	0.303
40	*MAE (within)*	0.128	0.402	0.843	0.164	0.254	0.408
	MAE (holdout)	0.251	0.875	0.599	0.384	0.989	0.115
	MAPE (within)	0.331	1.030	2.168	0.497	0.745	1.211
	MAPE (holdout)	0.593	2.068	1.419	1.001	2.579	0.302
65	*MAE (within)*	0.210	0.483	1.686	0.225	0.215	1.105
	MAE (holdout)	0.348	0.792	1.768	0.518	0.822	0.895
	MAPE (within)	1.206	2.720	9.437	1.575	1.538	7.587
	MAPE (holdout)	1.706	3.889	8.699	2.913	4.622	5.050
80	*MAE (within)*	0.107	0.123	4.335	0.093	0.312	3.757
	MAE (holdout)	0.125	0.083	4.997	0.216	0.090	4.311
	MAPE (within)	1.274	1.467	51.172	1.368	4.636	53.950
	MAPE (holdout)	1.259	0.834	50.685	2.545	1.069	51.142

Notes: MAPEs are in percent (%) and MAEs are in number of years. *Within* denotes the within-sample period and *holdout* denotes the holdout-sample period.

Table 6: 95% Confidence interval for forecasts of life expectancies

Panel A: US females

Year	e_0		e_20		e_40		e_65		e_80	
	Est.	95% CI	Est.	95% CI	Est.	95% CI	Est.	95% CI	Est.	95% CI
2006	80.37	(79.57, 81.20)	61.07	(60.69, 61.46)	41.78	(41.45, 42.14)	19.77	(19.51, 20.10)	9.57	(9.06, 9.59)
2007	80.55	(79.58, 81.54)	61.19	(60.71, 61.70)	41.90	(41.46, 42.36)	19.87	(19.49, 20.29)	9.63	(9.02, 9.69)
2008	80.76	(79.59, 81.94)	61.35	(60.76, 61.95)	42.03	(41.50, 42.60)	19.97	(19.51, 20.49)	9.70	(9.03, 9.84)
2009	80.96	(79.63, 82.28)	61.49	(60.82, 62.19)	42.16	(41.55, 42.81)	20.07	(19.54, 20.66)	9.77	(9.04, 9.96)
2010	81.16	(79.68, 82.64)	61.64	(60.89, 62.41)	42.30	(41.61, 43.02)	20.17	(19.58, 20.82)	9.84	(9.06, 10.09)
2011	81.36	(79.74, 82.98)	61.79	(60.97, 62.63)	42.43	(41.68, 43.22)	20.28	(19.62, 20.99)	9.91	(9.07, 10.21)
2012	81.56	(79.83, 83.31)	61.93	(61.05, 62.85)	42.56	(41.74, 43.42)	20.38	(19.68, 21.15)	9.98	(9.10, 10.32)
2013	81.77	(79.92, 83.63)	62.08	(61.14, 63.07)	42.69	(41.83, 43.61)	20.48	(19.73, 21.30)	10.05	(9.13, 10.44)
2014	81.98	(80.01, 83.96)	62.23	(61.23, 63.27)	42.83	(41.91, 43.81)	20.59	(19.79, 21.46)	10.12	(9.17, 10.55)
2015	82.18	(80.11, 84.30)	62.39	(61.33, 63.48)	42.96	(42.00, 43.99)	20.69	(19.85, 21.61)	10.19	(9.20, 10.66)
2016	82.39	(80.20, 84.61)	62.54	(61.43, 63.68)	43.10	(42.09, 44.19)	20.80	(19.91, 21.76)	10.27	(9.23, 10.77)
2017	82.60	(80.31, 84.93)	62.69	(61.53, 63.88)	43.24	(42.17, 44.37)	20.90	(19.98, 21.91)	10.34	(9.27, 10.89)
2018	82.81	(80.42, 85.25)	62.84	(61.64, 64.09)	43.38	(42.27, 44.56)	21.01	(20.05, 22.06)	10.41	(9.31, 11.00)
2019	83.02	(80.52, 85.54)	63.00	(61.74, 64.29)	43.51	(42.36, 44.75)	21.11	(20.11, 22.22)	10.49	(9.35, 11.11)
2020	83.24	(80.64, 85.85)	63.15	(61.85, 64.49)	43.65	(42.46, 44.94)	21.22	(20.18, 22.36)	10.56	(9.39, 11.22)
2021	83.45	(80.76, 86.18)	63.30	(61.97, 64.70)	43.79	(42.55, 45.13)	21.33	(20.26, 22.51)	10.64	(9.43, 11.33)
2022	83.67	(80.88, 86.49)	63.46	(62.08, 64.90)	43.94	(42.65, 45.31)	21.43	(20.33, 22.66)	10.72	(9.49, 11.44)
2023	83.89	(81.00, 86.80)	63.62	(62.19, 65.10)	44.08	(42.76, 45.50)	21.54	(20.41, 22.80)	10.80	(9.53, 11.55)
2024	84.11	(81.13, 87.11)	63.77	(62.31, 65.30)	44.22	(42.87, 45.68)	21.65	(20.48, 22.95)	10.87	(9.57, 11.66)
2025	84.33	(81.26, 87.44)	63.93	(62.43, 65.49)	44.36	(42.97, 45.86)	21.76	(20.56, 23.09)	10.95	(9.62, 11.77)
2026	84.55	(81.38, 87.75)	64.09	(62.54, 65.69)	44.51	(43.07, 46.05)	21.87	(20.64, 23.24)	11.03	(9.68, 11.88)
2027	84.77	(81.49, 88.08)	64.25	(62.67, 65.89)	44.66	(43.18, 46.23)	21.98	(20.72, 23.38)	11.11	(9.73, 12.00)
2028	84.99	(81.62, 88.39)	64.41	(62.78, 66.09)	44.80	(43.29, 46.42)	22.09	(20.80, 23.54)	11.19	(9.78, 12.12)
2029	85.22	(81.76, 88.70)	64.57	(62.92, 66.29)	44.95	(43.40, 46.60)	22.20	(20.89, 23.68)	11.28	(9.84, 12.23)
2030	85.45	(81.90, 89.06)	64.73	(63.03, 66.49)	45.10	(43.52, 46.80)	22.31	(20.97, 23.83)	11.36	(9.88, 12.34)
2031	85.67	(82.03, 89.37)	64.90	(63.17, 66.68)	45.25	(43.63, 46.98)	22.43	(21.06, 23.98)	11.44	(9.94, 12.45)
2032	85.90	(82.17, 89.68)	65.06	(63.29, 66.89)	45.40	(43.76, 47.16)	22.54	(21.14, 24.14)	11.52	(10.00, 12.57)
2033	86.13	(82.32, 90.03)	65.22	(63.42, 67.09)	45.55	(43.87, 47.35)	22.66	(21.24, 24.28)	11.61	(10.06, 12.69)
2034	86.37	(82.46, 90.35)	65.39	(63.54, 67.29)	45.70	(43.99, 47.54)	22.77	(21.32, 24.44)	11.69	(10.11, 12.79)
2035	86.60	(82.58, 90.68)	65.55	(63.68, 67.48)	45.86	(44.11, 47.73)	22.89	(21.41, 24.59)	11.78	(10.17, 12.91)
2036	86.84	(82.72, 91.03)	65.72	(63.82, 67.70)	46.01	(44.24, 47.92)	23.01	(21.51, 24.73)	11.86	(10.23, 13.02)
2037	87.07	(82.88, 91.37)	65.89	(63.95, 67.89)	46.17	(44.35, 48.11)	23.12	(21.59, 24.88)	11.95	(10.29, 13.14)
2038	87.31	(83.01, 91.70)	66.05	(64.08, 68.10)	46.32	(44.48, 48.30)	23.24	(21.69, 25.04)	12.03	(10.35, 13.26)
2039	87.55	(83.17, 92.02)	66.22	(64.22, 68.30)	46.48	(44.61, 48.49)	23.36	(21.78, 25.19)	12.12	(10.41, 13.38)
2040	87.79	(83.32, 92.38)	66.39	(64.35, 68.52)	46.64	(44.73, 48.68)	23.48	(21.88, 25.34)	12.21	(10.48, 13.50)
2041	88.03	(83.46, 92.73)	66.56	(64.49, 68.71)	46.80	(44.85, 48.87)	23.60	(21.97, 25.50)	12.29	(10.54, 13.61)
2042	88.28	(83.60, 93.07)	66.73	(64.63, 68.91)	46.96	(44.98, 49.06)	23.73	(22.07, 25.65)	12.38	(10.61, 13.73)
2043	88.53	(83.76, 93.39)	66.90	(64.77, 69.12)	47.12	(45.10, 49.26)	23.85	(22.17, 25.81)	12.47	(10.67, 13.86)
2044	88.77	(83.92, 93.76)	67.08	(64.91, 69.33)	47.29	(45.25, 49.46)	23.98	(22.27, 25.97)	12.56	(10.74, 13.98)

2045	89.02	(84.08, 94.11)	67.25	(65.05, 69.53)	47.45	(45.37, 49.66)	24.10	(22.38, 26.12)	12.65	(10.81, 14.10)
2046	89.27	(84.22, 94.41)	67.42	(65.21, 69.74)	47.62	(45.51, 49.86)	24.23	(22.47, 26.28)	12.74	(10.87, 14.22)
2047	89.52	(84.37, 94.82)	67.60	(65.36, 69.95)	47.78	(45.64, 50.06)	24.36	(22.57, 26.45)	12.83	(10.94, 14.34)
2048	89.78	(84.52, 95.16)	67.78	(65.50, 70.15)	47.95	(45.78, 50.26)	24.49	(22.67, 26.61)	12.92	(11.02, 14.47)
2049	90.03	(84.69, 95.51)	67.95	(65.65, 70.35)	48.12	(45.92, 50.45)	24.62	(22.78, 26.77)	13.01	(11.09, 14.59)
2050	90.29	(84.85, 95.89)	68.13	(65.79, 70.55)	48.29	(46.06, 50.66)	24.75	(22.89, 26.94)	13.10	(11.16, 14.71)

Panel B: US males

Year	e_0		e_20		e_40		e_65		e_80	
	Est.	*95% CI*	*Est.*	*95% CI*	*Est.*	*95% CI*	*Est.*	*95% CI*	*Est.*	*95% CI*
2006	75.33	(73.45, 77.23)	56.23	(55.49, 56.96)	37.66	(37.14, 38.21)	17.04	(16.65, 17.45)	8.10	(7.48, 8.15)
2007	75.63	(72.88, 78.48)	56.40	(55.28, 57.50)	37.79	(37.01, 38.62)	17.13	(16.59, 17.74)	8.16	(7.40, 8.29)
2008	75.93	(72.51, 79.51)	56.58	(55.14, 57.97)	37.92	(36.94, 38.99)	17.23	(16.56, 18.01)	8.22	(7.36, 8.46)
2009	76.24	(72.25, 80.52)	56.77	(55.05, 58.41)	38.05	(36.90, 39.33)	17.33	(16.56, 18.25)	8.28	(7.34, 8.62)
2010	76.55	(72.03, 81.46)	56.95	(54.98, 58.84)	38.19	(36.87, 39.66)	17.43	(16.58, 18.46)	8.35	(7.33, 8.77)
2011	76.87	(71.86, 82.39)	57.15	(54.94, 59.25)	38.33	(36.86, 40.00)	17.52	(16.60, 18.69)	8.41	(7.32, 8.92)
2012	77.19	(71.78, 83.26)	57.34	(54.91, 59.66)	38.47	(36.84, 40.33)	17.62	(16.64, 18.91)	8.48	(7.33, 9.06)
2013	77.52	(71.68, 84.15)	57.55	(54.89, 60.08)	38.62	(36.84, 40.68)	17.72	(16.67, 19.13)	8.54	(7.33, 9.22)
2014	77.86	(71.63, 85.09)	57.75	(54.89, 60.49)	38.77	(36.84, 41.04)	17.82	(16.72, 19.36)	8.61	(7.36, 9.36)
2015	78.20	(71.59, 86.03)	57.96	(54.86, 60.88)	38.93	(36.85, 41.38)	17.91	(16.77, 19.58)	8.68	(7.38, 9.52)
2016	78.55	(71.53, 86.93)	58.18	(54.87, 61.30)	39.09	(36.87, 41.74)	18.01	(16.82, 19.81)	8.75	(7.40, 9.67)
2017	78.90	(71.53, 87.88)	58.39	(54.87, 61.72)	39.26	(36.87, 42.10)	18.11	(16.88, 20.04)	8.82	(7.43, 9.83)
2018	79.27	(71.51, 88.82)	58.62	(54.89, 62.15)	39.43	(36.90, 42.49)	18.21	(16.94, 20.30)	8.89	(7.44, 10.01)
2019	79.64	(71.51, 89.74)	58.84	(54.89, 62.56)	39.60	(36.90, 42.86)	18.31	(17.00, 20.55)	8.96	(7.47, 10.19)
2020	80.01	(71.53, 90.66)	59.07	(54.89, 62.99)	39.78	(36.90, 43.27)	18.41	(17.06, 20.81)	9.03	(7.51, 10.37)
2021	80.40	(71.52, 91.72)	59.31	(54.92, 63.42)	39.97	(36.91, 43.67)	18.51	(17.13, 21.07)	9.10	(7.54, 10.56)
2022	80.79	(71.56, 92.68)	59.55	(54.94, 63.86)	40.16	(36.92, 44.09)	18.61	(17.18, 21.35)	9.17	(7.58, 10.77)
2023	81.19	(71.59, 93.75)	59.79	(54.96, 64.31)	40.36	(36.94, 44.50)	18.72	(17.25, 21.64)	9.24	(7.62, 10.98)
2024	81.59	(71.62, 94.79)	60.04	(55.02, 64.77)	40.56	(36.96, 44.93)	18.82	(17.32, 21.95)	9.31	(7.67, 11.21)
2025	82.01	(71.70, 95.89)	60.29	(55.04, 65.23)	40.77	(36.99, 45.35)	18.93	(17.38, 22.27)	9.37	(7.71, 11.44)
2026	82.43	(71.70, 97.05)	60.54	(55.07, 65.67)	40.98	(37.02, 45.80)	19.05	(17.45, 22.60)	9.44	(7.77, 11.69)
2027	82.86	(71.77, 98.16)	60.80	(55.12, 66.11)	41.20	(37.03, 46.24)	19.16	(17.52, 22.93)	9.51	(7.82, 11.96)
2028	83.30	(71.81, 99.35)	61.07	(55.15, 66.58)	41.43	(37.06, 46.70)	19.28	(17.59, 23.30)	9.58	(7.86, 12.24)
2029	83.75	(71.88, 100.49)	61.33	(55.18, 67.05)	41.66	(37.07, 47.16)	19.41	(17.66, 23.65)	9.64	(7.92, 12.51)
2030	84.20	(71.95, 101.69)	61.61	(55.22, 67.50)	41.90	(37.13, 47.60)	19.54	(17.73, 24.00)	9.71	(7.97, 12.80)
2031	84.67	(72.03, 102.92)	61.88	(55.29, 67.94)	42.15	(37.16, 48.05)	19.67	(17.81, 24.36)	9.78	(8.02, 13.10)
2032	85.14	(72.07, 104.16)	62.16	(55.33, 68.42)	42.40	(37.16, 48.50)	19.81	(17.88, 24.73)	9.84	(8.07, 13.38)
2033	85.62	(72.12, 105.58)	62.45	(55.36, 68.87)	42.66	(37.18, 48.98)	19.96	(17.96, 25.10)	9.91	(8.14, 13.66)
2034	86.12	(72.25, 106.95)	62.74	(55.39, 69.32)	42.92	(37.18, 49.43)	20.12	(18.01, 25.46)	9.98	(8.20, 13.95)
2035	86.62	(72.33, 108.32)	63.03	(55.48, 69.75)	43.19	(37.22, 49.86)	20.28	(18.10, 25.83)	10.05	(8.27, 14.26)
2036	87.13	(72.39, 109.71)	63.33	(55.52, 70.20)	43.47	(37.24, 50.25)	20.45	(18.16, 26.20)	10.13	(8.32, 14.57)
2037	87.65	(72.49, 111.06)	63.63	(55.59, 70.60)	43.75	(37.25, 50.68)	20.62	(18.24, 26.57)	10.21	(8.38, 14.84)
2038	88.19	(72.57, 112.57)	63.93	(55.67, 71.05)	44.04	(37.28, 51.12)	20.81	(18.31, 26.98)	10.29	(8.45, 15.13)
2039	88.73	(72.70, 113.95)	64.24	(55.73, 71.48)	44.34	(37.30, 51.52)	21.00	(18.38, 27.35)	10.37	(8.52, 15.43)
2040	89.28	(72.82, 115.73)	64.56	(55.80, 71.91)	44.64	(37.38, 51.95)	21.20	(18.45, 27.73)	10.46	(8.59, 15.73)
2041	89.85	(72.89, 117.41)	64.87	(55.89, 72.34)	44.95	(37.36, 52.38)	21.41	(18.52, 28.10)	10.56	(8.65, 16.02)
2042	90.43	(72.95, 119.01)	65.20	(55.96, 72.74)	45.26	(37.40, 52.77)	21.63	(18.59, 28.47)	10.66	(8.72, 16.29)
2043	91.01	(73.14, 120.62)	65.53	(56.02, 73.14)	45.58	(37.43, 53.16)	21.86	(18.67, 28.86)	10.78	(8.79, 16.57)
2044	91.61	(73.23, 122.21)	65.86	(56.12, 73.52)	45.90	(37.47, 53.59)	22.09	(18.75, 29.22)	10.89	(8.85, 16.86)
2045	92.22	(73.36, 123.82)	66.20	(56.18, 73.93)	46.23	(37.48, 53.99)	22.34	(18.81, 29.57)	11.02	(8.93, 17.12)
2046	92.85	(73.46, 125.68)	66.54	(56.25, 74.34)	46.57	(37.56, 54.37)	22.60	(18.86, 29.91)	11.16	(9.01, 17.40)

2047	93.49	(73.55, 127.43)	66.89	(56.33, 74.72)	46.91	(37.57, 54.74)	22.87	(18.94, 30.26)	11.31	(9.08, 17.66)
2048	94.14	(73.68, 129.04)	67.24	(56.40, 75.10)	47.26	(37.60, 55.14)	23.14	(19.00, 30.61)	11.47	(9.15, 17.91)
2049	94.80	(73.83, 130.76)	67.59	(56.46, 75.49)	47.61	(37.67, 55.54)	23.43	(19.07, 30.97)	11.64	(9.24, 18.17)
2050	95.47	(73.94, 132.85)	67.96	(56.51, 75.85)	47.97	(37.64, 55.93)	23.73	(19.16, 31.29)	11.82	(9.33, 18.45)