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Heterogeneity of Capital Stocks in Japan: Classification by Factor Analysis[*]

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Abstract

This paper examines the heterogeneity of capital stocks using financial statement data of publicly listed Japanese firms. We conduct factor analysis on investment rates among various capital goods and estimate factor loadings of each as its reactions to common factors like total factor productivity (TFP) shocks. Then we estimate the uniqueness for each investment rate, which is the percentage of its variance that is not explained by the common factors. If the estimated factor loadings are similar between some of the heterogeneous capital goods, it may well imply that the adjustment cost structure of these investments is also similar. Further, if some of the estimated values of uniqueness are small, it suggests that certain theoretical models may track the dynamics of the investment rates well.

Our estimation results show that Building and Structure have similar factor loadings as do Machinery & Equipment, Vehicles & Delivery Equipment, and Tools, Furniture, & Fixture. This suggests that we could remedy the Curse of Dimensionality by bundling the investments that have similar factor loadings together and that identifying the functional structures of each group of capital goods can greatly improve the performance of empirical investment equations.

Keywords: Investment, Heterogeneity of Capital Stocks, Investment Purchase and Sale, Factor Analysis, Adjustment Costs, Tobin's Q.

JFL classification: D21, D92, E22, E32

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1. Introduction

Tobin's q (market value of installed capital/replacement cost of capital) is one of the most popular theories used for conducting empirical research on capital investment. It was presented by Tobin in 1969 and was linked to neoclassical economic investment models by introducing a convex adjustment cost of investment.[1] Although this theory has solid microeconomic foundations, its empirical performance has been disappointing. Asako and Kuninori (1989) summarized the problems in the empirical performance of the q model based on publicly listed Japanese firms as follows:

1. Tobin's q, which is considered a sufficient statistic for investment decisions, has little explanatory power.
2. Cash flow, output, operating ratio, etc. are significant, and adding these variables reduces the explanatory power of Tobin's q.
3. Autocorrelation of disturbances exists. Moreover, historical Tobin's q values are significant if they are included in explanatory variables.

Since the second half of the 1980s, researchers have tried to find the causes of the low empirical performance of Tobin's q theory and attempted to overcome the problems. Erickson and Whited (2000) classified the causes in the following three categories:

1. The idea that owner-managers decide the investment amount solely on the basis of their expectations about future profits is not consistent with actual observations.
2. The econometric assumptions used to derive linear investment functions of Tobin's q are not correct. Endogeneity between Tobin's q and the investment rate, nonlinear investment functions, etc. should be considered.
3. Average q (original Tobin's q) is not sufficient as a proxy variable to marginal q, which has robust neoclassical micro foundations, on account of measurement errors.

This list seems comprehensive, but fails to consider the heterogeneity in the adjustment costs of investments.

[1] See Lucas and Prescott (1971); Mussa (1977); Nickell (1978); Abel (1980); Yoshikawa (1980); and Hayashi (1982).

Cooper and Haltiwanger (2006) discussed the most general structure of the adjustment costs of total investment, incorporating all types of investment models that had appeared since the Tobin's q theory. The structure consists of non-convex costs in addition to convex adjustment costs, e.g., fixed cost, loss of profit during installation, and investment irreversibility. They estimated their parameters using plant-level data of the U.S. manufacturing sector via simulated method of moment (SMM), and their model performed far better than earlier models. They stated that the comprehensive structure of adjustment costs would have been most desirable because in reality there existed many types of capital goods, each with their own structure of adjustment cost.

Wildasin (1984) first considered the heterogeneity of capital stocks under the Tobin's q framework. The empirical works using financial statement data of listed Japanese firms have since followed (Asako, Kuninori, Inoue, and Murase, 1989, 1997). They referred to Tobin's q as "Multiple q"when the heterogeneity of capital stocks was considered and as "Single q" when the heterogeneity of capital stocks was not considered. Recently Tonogi, Nakamura, and Asako (2010), Asako and Tonogi (2010), and Asako, Tonogi, and Nakamura (2013) explored "Multiple q" investment equations based on a more detailed classification of capital stocks. They showed that "Multiple q" exhibited better fitness than "Single q," but the explanatory power did not improve significantly enough even by considering the heterogeneity of capital stocks. They also tried to identify ranges of investment rates where a parameter of convex adjustment cost for each investment rate was insignificant and demonstrated that non-convex adjustment costs were present. These empirical results and the study by Cooper and Haltiwanger (2006) suggest that it is important to consider the heterogeneity of capital stocks and comprehensive structures of adjustment costs simultaneously.

In this paper, we implement the method of factor analysis on various investment rates to examine the heterogeneity of capital stocks. Our data consist of seven capital goods: [1] Building, [2] Structure, [3] Machinery & Equipment, [4] Vehicles & Delivery Equipment, [5] Shipment, [6] Tools, Furniture, & Fixture, and [7] Land, constructed from financial statement data of listed Japanese firms. We estimate factor loadings as reactions to common factors among the various capital stocks such as total factor productivity (TFP) shocks and classify the investments depending on whether or not they have similar factor loadings. If some of the estimated factor loadings are similar, their investment dynamics are also considered to be similar. Consequently, we can say analogously that the parameters for adjustment costs of these investments should be similar, without specifying the functional structures of the adjustment costs.

Asako and Tonogi (2010) examined the hypotheses that the parameters of convex adjustment costs are the same in all combinations of investments and obtained no consistent combinations of investments among all of their sample periods and all the ways of data construction. However, this result may be caused by the assumption of convex adjustment costs. Therefore, in this paper, we reexamine the heterogeneity of capital stocks via the factor analysis without assuming any specific adjustment cost structure.

Our approach may seem roundabout, and it could be criticized that we should model comprehensive structures of capital adjustment costs under heterogeneous capital stocks and estimate their parameters immediately. However, if done in this manner, the Curse of Dimensionality[2] would emerge because we would identify dozens of parameters for seven types of capital goods with several types of adjustment costs in each. We believe it is necessary to reduce the number of parameters before structural estimation of capital adjustment costs under heterogeneous capital stocks.

Through the factor analysis on the seven types of investment rates, we find that [1] Building and [2] Structure have similar factor loadings. Besides this pair, [3] Machinery & Equipment, [4] Vehicles & Delivery Equipment, and [6] Tools, Furniture, & Fixture also have similar factor loadings. By bundling the investments that have similar factor loadings together, we could alleviate the Curse of Dimensionality. We find that the values of uniqueness of the factor analysis for [1] Building and [2] Structure are low, which shows that these investments can be explained fairly well by some theoretical model. Even though including [5] Shipment and [7] Land was rarely explained by common factors, such a theoretical model would track the dynamics of all the investments much better than models that consider just the total investment.

We organize the rest of this paper as follows. Section 2 introduces our estimation methods, which incorporate reviews of various investment models and basics of factor analysis. Section 3 discusses our data-set and the empirical results of our analysis. Section 4 presents the conclusion.

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[2] The curse of dimensionality refers to problems that arise while conducting data analysis in high-dimensional spaces; such problems typically do not occur in low-dimensional settings.

2. Estimation Methods

In this section, we discuss how to conduct the factor analysis on investment rates. In 2.1, we review investment models, while in 2.2, we describe a basic model of factor analysis to estimate investment dynamics driven by common shocks (components of TFP) and touch upon how to classify heterogeneous investments. We specify a rotation method of factor loadings in our analysis in 2.3, and a method of factor analysis and how to determine the number of factors in 2.4.

2.1 Investment Models

We start with "Single q," a Tobin's q model that considers just the total investment assuming the homogeneity of capital stocks and extend it to "Multiple q," one that considers the heterogeneity of capital stocks. We then show that all the investment rates are driven by the same TFP shocks, the components of which are estimated as common shocks of factor analysis among the investment rates. The fact that all the investment rates are driven by common shocks does not change even if we consider other sorts of adjustment cost in addition to the convex adjustment cost.

Let us consider a following profit maximization problem about the total investment with a convex adjustment cost:

$$V(A,K) = \max_{K'} \left[AK^{\alpha} - \frac{\gamma}{2}\left(\frac{K' - (1-\delta)K}{K}\right)^2 K - p(K' - (1-\delta)K) + \beta E_{A'|A}\{V(A',K')\} \right] \tag{1}$$

where $A, K, \delta, p,$ and β represent a TFP shock, the total capital stock, the depreciation rate of capital stock, the price of investment, and the discount factor, respectively. Variables with prime symbols refer to variables in the next period. E stands for the operator of taking the expected values and thereby $E_{A'|A}(X)$ implies that the variable X is evaluated at that level where A' takes the expected value conditional upon A.

Denoting by I the level of investment, derived as $I = K' - (1-\delta)K$, the first-order condition (F.O.C) of (1) derives a linear function of Tobin's q[3]:

$$\frac{I}{K} = \frac{1}{\gamma}\left(\beta E_{A'|A}\{V_{K'}(A',K')\} - p\right)$$

$$= \frac{1}{\gamma} E_{A'|A}[(q-1)p] \quad \text{where } q = \beta E_{A'|A}\frac{[V]}{pK'} \tag{2}$$

because the profit function and the convex adjustment cost function in (1) exhibit the characteristics of being the homogeneous of degree one (or constant returns to scale) and hence $E_{A'|A}\{V'_{K'}\} = E_{A'|A}\left\{{V'}/{K'}\right\}$, as shown by Hayashi (1982). This is a "Single q" investment equation.

Cooper and Haltiwanger (2006) discussed the most general structure of adjustment costs of total investment, incorporating all types of investment models that had appeared since the Tobin's q theory:

$$V(A,K) = max\{V^b(A,K), V^s(A,K), V^i(A,K)\} \tag{3}$$

$$\text{where} \quad V^b(A,K) = \max_{K'} \left[\mu A K^\alpha - FK - \frac{\gamma}{2}\left(\frac{K' - (1-\delta)K}{K}\right)^2 K - p_b(K' - (1-\delta)K) + \beta E_{A'|A}\{V(A',K')\}\right],$$

$$V^s(A,K) = \max_{K'} \left[\mu A K^\alpha - FK - \frac{\gamma}{2}\left(\frac{K' - (1-\delta)K}{K}\right)^2 K - p_s(K' - (1-\delta)K) + \beta E_{A'|A}\{V(A',K')\}\right], \quad \text{and}$$

$$V^i(A,K) = AK^\alpha + \beta E_{A'|A}\{V(A',(1-\delta)K)\}]$$

$$\text{with } 0 \le \mu \le 1, \quad F \ge 0, \quad p_s/p_b \le 1.$$

[3] To be precise, the discounted present value of the next period's Tobin's q.

The dynamics of investment changes from (1) to (3) to incorporate various adjustment costs, but the fact that it is driven by TFP shocks does not change.

If we consider heterogeneity of capital stocks in (1), the profit maximization problem becomes:

$$V(A, K_1, \cdots, K_n) = \max_{K'_j}[AK_1^{\alpha_1} \cdots K_n^{\alpha_n} - \sum_{j=1}^{n} \frac{\gamma_j}{2}\left(\frac{K_j' - (1-\delta_j)K_j)}{K_j}\right)^2 K_j \quad (4)$$

$$- \sum_{j=1}^{n} p_j\big(K'_j - (1-\delta_j)K_j\big) + \beta E_{A'|A}\{V(A', K'_1, \cdots, K'_n)\}]$$

where the subscript j represents types of investment goods and $\sum_j a_j = 1$.[4] The F.O.C of (4) for each capital stock is:

$$\frac{I_j}{K_j} = \frac{1}{\gamma_j}\big(\beta E_{A'|A}\big[\partial V(A', K_1', \cdots, K_n')/\partial K_j'\big] - p_j\big)$$

$$= \frac{1}{\gamma_j}\big(q_j - 1\big)p_j \quad \text{where } q_j = \frac{\beta E_{A'|A}[V_{K_j}(A', K_1', \cdots, K_n')]}{p_j}. \quad (5)$$

Tobin's q for each capital stock, q_j has been called "Partial q" since the study by Asako, Kuninori, Inoue, and Murase (1989). The value function is the homogeneous of degree one in capital stocks; hence the Euler theorem allows us to sum up (5) in the following manner:

$$\sum_{j=1}^{n} \frac{\partial V(A, K_1, \cdots, K_n)}{\partial K_j} K_j = V(A, K_1, \cdots, K_n).$$

Then, from the above, we obtain:

[4] We also assume that the adjustment costs of the investments are expressed to be additive.

$$(q-1)P = \sum_{j=1}^{n} \gamma_j \left(\frac{K' - (1-\delta)K}{K} \right) s_j \qquad (6)$$

$$\text{where} \begin{cases} q = \dfrac{\beta V'}{\sum_{j=1}^{n} p_j K_j{}'} \\ P = \dfrac{\sum_{j=1}^{n} p_j K_j{}'}{\sum_{j=1}^{n} K_j{}'} = \sum_{j=1}^{n} p_j s_j \\ s_j = \dfrac{K_j{}'}{\sum_{j=1}^{n} K_j{}'} \end{cases}$$

This is a "Multiple q" investment equation. Even if we consider various adjustment costs like (3) for each investment good, the fact that all the investment rates are driven by the TFP shocks unchanges, as shown in (5).

If all γ_j are equal in (6), the following "Single q" equation obtains:

$$(q-1)P = \gamma \left(\frac{\bar{I}}{\bar{K}} \right) \text{ where } \frac{\bar{I}}{\bar{K}} = \sum_{j=1}^{n} \left(\frac{I_j}{K_j} \right) s_j \text{ and } \gamma_j = \gamma \ \forall j \qquad (7)$$

It is worth highlighting that we can treat various capital stocks as one if their parameters of adjustment costs are equal as with (7), even though we introduce various adjustment costs in (3) for each capital stock.

2.2 Basic Factor Model

Our basic factor model is as follows:

$$z_{ij} = a_{j1} f_{i1} + a_{j2} f_{i2} + \cdots + a_{jm} f_{im} + d_j u_{ij} \qquad (8)$$

where z_{ij} is the investment rate for each i ($i = 1,2,\cdots,N$) firm and each j ($j = 1,2,\cdots,n$) investment good.[5] $f_{i1}, f_{i2}, \cdots, f_{im}$ are m common factors, e.g., components of TFP shocks, which stimulate all investment rates for i firm, while u_{ij} represents the individual factors for j investment of i firm. a_{jp} is referred to as a factor loading, which indicates a reaction of j investment to common factor p, while d_j is the weight of the individual factor of j investment rate. a_{jp} and d_j for j investment are common among all firms. A graphical representation of this factor model is depicted in Figure 1.

If we apply the factor analysis with only one common factor to investment rates whose dynamics are driven by the Multiple q framework, the common factor corresponds to Partial q driven by the TFP shock A which is common among various investment goods, and each factor loading corresponds to a parameter of the convex adjustment cost for each capital. The precise relationship between equations (5) and (8) is derived in the Appendix. If some of the investment rates have the same parameter values for the adjustment costs, their reactions to the Partial q and equivalently their factor loadings should also be the same. Whatever types of adjustment cost other than convex costs are introduced,[6] factor loadings correspond to parameters of the adjustment costs. In this paper, we classify capital stocks into groups using their factor loadings.[7]

Equation (1) can be rewritten as a Matrix form:

$$\boldsymbol{Z} = \boldsymbol{F}\boldsymbol{A}' + \boldsymbol{U}\boldsymbol{D} \tag{9}$$

$$(N \times n) = (N \times m) \cdot (n \times m)' + (N \times n)(n \times n)$$

where

[5] It is standardized with zero mean and unit variance.

[6] Nonlinear adjustment costs are assumed in equation (A5) in the Appendix.

[7] In econometric analysis using factor models, a basic model is often $z_{ti} = a_{i1}f_{t1} + a_{i2}f_{t2} + \cdots + a_{im}f_{tm} + d_{ti}$, where i represents firm ID or variable ID and t represents period. In this case, the common factors comprise some macro shocks and the factor loadings are reactions to them. It is worth noting that the role of ID is opposite to our model.

$$\boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & & \vdots \\ z_{N1} & z_{N2} & \cdots & z_{Nn} \end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1m} \\ f_{21} & f_{22} & \cdots & f_{2m} \\ \vdots & \vdots & & \vdots \\ f_{N1} & f_{N2} & \cdots & f_{Nm} \end{bmatrix}, \ \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix},$$

$$\boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \vdots & \vdots & & \vdots \\ u_{N1} & u_{N2} & \cdots & u_{Nn} \end{bmatrix}, \qquad \boldsymbol{D} = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & d_n \end{bmatrix}$$

We introduce the following four standard assumptions in conducting the factor analysis in this paper. First, the averages of common factors and individual factors are assumed to be zero:

$$\boldsymbol{F}'\boldsymbol{1} = \boldsymbol{0}, \ \ \boldsymbol{U}'\boldsymbol{1} = \boldsymbol{0}\,. \qquad (10)$$

Second, a correlation matrix among common factors is written as

$$\frac{1}{N}\boldsymbol{F}'\boldsymbol{F} = \boldsymbol{L} = \begin{bmatrix} 1 & l_{12} & \cdots & l_{1m} \\ l_{21} & 1 & \cdots & l_{2m} \\ \vdots & \vdots & & \vdots \\ l_{m1} & l_{m2} & \cdots & 1 \end{bmatrix} \qquad (11)$$

where the diagonal elements are all 1. Third, non-diagonal elements in equation (11) are all zeros, where the model is called an “orthogonal factor model.”[8] Fourth, common and individual factors have no correlation, and individual factors are assumed to be orthogonal:

$$\boldsymbol{F}'\boldsymbol{U} = \boldsymbol{0}, \quad \frac{1}{N}\boldsymbol{U}'\boldsymbol{U} = \boldsymbol{I_n} \qquad (12)$$

[8] A model is called an “oblique factor model” when not all non-diagonal elements in equation (11) are zero.

Under these assumptions, the correlation matrix can be decomposed as follows:

$$\boldsymbol{R} = \frac{1}{N}\boldsymbol{Z'Z}$$

$$= \frac{1}{N}\boldsymbol{AF'FA'} + \frac{1}{N}\boldsymbol{AF'UD} + \frac{1}{N}\boldsymbol{DU'FA'} + \frac{1}{N}\boldsymbol{DU'UD}$$

$$= \boldsymbol{ALA'} + \boldsymbol{D'D}$$

$$= \boldsymbol{AA'} + \boldsymbol{D'D}$$

where the diagonal elements are:

$$\sum_{p=1}^{m} a_{jp}^2 + d_j^2$$

which means that the variance of each investment rate standardized to 1 can be decomposed into "communality," which a common factor can explain, and "uniqueness," which it cannot.[9] If the values of uniqueness are high, any theoretical model, even one with comprehensive adjustment costs, can hardly replicate the dynamics of investment rates.

2.3 Rotation of Common Factors

Factor analysis condenses the dynamics of many variables into fewer factors. Factor loadings represent reactions to common factors; it is important to note that "reactions" do not mean actual causalities. After estimating factors, we can explore *ex post facto* relations between estimated factors and actual exogenous causes.

[9] In other words, communality and uniqueness correspond to the contribution rates to not standardized variances of common factors and residual factors, respectively .

We can obtain other expressions such as:

$$\boldsymbol{Z} = \boldsymbol{G}\boldsymbol{B}' + \boldsymbol{U}\boldsymbol{D}$$

by transforming the factor matrix $\boldsymbol{F}$ by any $\boldsymbol{T}$ to $\boldsymbol{G} = \boldsymbol{F}\boldsymbol{T}$ and defining new matrix factor loadings $\boldsymbol{B}$ corresponding to $\boldsymbol{T}$. Further, there exist innumerable combinations of $\boldsymbol{G}$ and $\boldsymbol{B}$. In this paper, we rotate estimated common factors by the method of "orthogonal varimax rotation" in a simple structure, where each investment rate reacts strongly to one of the rotated common factors and weakly to others. Then we classify investments into groups, each of which has the same strong reaction to one of the factors.[10]

2.4 Estimation Methods and Determination of the Number of Common Factors

Factor analysis and principal component analysis are often confused. The principal components in principal component analysis are synthetic variables composed of observation variables, while factors in factor analysis compose observation variables. Chamberlain and Rothschild (1983) showed that the result of eigenvalue analysis of factor models was the same as principal component analysis asymptotically when $n \to \infty$. Since then, other methods using principal component analysis instead of factor analysis have been developed. Connor and Korajzcyk (1986, 1988) first considered using asymptotic principal components as estimators of factors when N is fixed and $n \to \infty$. Bai and Ng (2008) studied the estimators when $n \to \infty$ and $N \to \infty$. In our analysis, because n is fixed as the number of investments, principal component analysis would not be appropriate. Therefore, we adopt the principal factor method as a conventional method of factor analysis.

[10] Factor models can be written in a vector expression: $\boldsymbol{z_j} = a_j\boldsymbol{f_1} + a_2\boldsymbol{f_2} + \cdots + a_{jm}\boldsymbol{f_m} + d_j\boldsymbol{u_j}$, where $\boldsymbol{f_1}, \boldsymbol{f_2}, \cdots, \boldsymbol{f_m}$ are fundamental vectors in common factor space. With vectors of factor loadings in geometrical representation, it is notable that their length and the angles between them are determined by their communalities and correlations of $\boldsymbol{z_j}$s and that these are not changed by any rotations.

We refer to Thurstone (1947) to determine the number of factors. In general, $m \times n$, which is the number of temporal factors, should be smaller than ${}_{\mathrm{n}}\mathrm{C}_2 = n(n-1)/2$, which represents the number of elements in a correlation matrix for n variables. Then, the following must be satisfied:

$$\frac{n(n-1)}{2} \geq nm - \frac{m(m-1)}{2} \;\; \Rightarrow \;\; m \leq \frac{(2n+1) - \sqrt{8n+1}}{2}$$

where an assumption that there is no correlation between m factors reduces the number of independent variables by $\frac{m(m-1)}{2}$. In our analysis, we treat seven capital goods and then $m \leq 3.73$ with $n = 7$. In addition, we determine the number of factors, which should be an integer, to avoid inadequate factor loadings that are imaginary. Consequently, in all the following estimations, the number of factors is determined as 3.

3. Data Management and Empirical Results

In this section, we estimate the factor model on the investment rates of listed Japanese firms using their financial statements. First we summarize the data construction issues in 3.1 and then discuss the sample period and outliers in 3.2. Subsequently, 3.3 confirms our estimation model and 3.4 reports the results. Since we completely follow the method of Tonogi, Nakamura, and Asako (2010) to construct capital stock and investment data, refer to the original paper for further details.

3.1 Construction of Capital Stocks and Investment Series

The data used in our analysis are constructed from “DBJ Financial Database of Listed Firms” released by the Development Bank of Japan, which contains individual firms’ financial statement data listed in the First and Second Sections of the Tokyo, Osaka, and Nagoya Stock Exchanges. The data series are extended to FY 2010 in this paper, while it culminated in FY 2007 in Tonogi, Nakamura and Asako (2010). Our panel data-set is unbalanced one, as it contains delisted firms and newly listed ones. The capital stock series are constructed by the perpetual inventory method using 1977 or the first recorded year after 1977 in the “DBJ Financial Database of Listed Firms” as a benchmark year for each firm. The database contains detailed data of depreciable assets by 6 items: [1] Building, [2] Structure, [3] Machinery & Equipment, [4] Vehicles & Delivery Equipment, [5] Shipment, and [6] Tools, Furniture & Fixture. We compute investment rates for each of 6 items as well as those for [7] Land. That is, the number of investment series is seven.

There are two concepts regarding investments and capital stock statistics: “in-progress,” which treat expenditures as investments when they are expensed and “installation,” which treat expenditures as investments after they begin operating to produce. We adopt “installation” and calculate investments according to the following formula:

Net Investment = Purchased Investment - Sold and Retired Investment

However, current values of the sold and retired investments are unobservable and we have to estimate them by using other data in the financial statements.

In previous studies, we find that researchers used three ways to treat this issue. The first one is that the book value of sold and retired investments is calculated back by definitional identity equations in accounting; these are then multiplied by the current value/book value ratios estimated under certain assumptions about depreciation schedules. We call this “Proportional way,” which has been adopted by Asako, Kuninori, Inoue, and Murase (1989) and Hayashi and Inoue (1991). The second one is that the book value of sold and retired investments is used as their current value to avoid the overestimation tendency inherent in the Proportional way. We call this “Book-value way,” which has been adopted by Suzuki (2001). The third one is that current values of sold and retired investments are all zero based on the view that it is impossible to estimate current values of the sold and retired investments correctly. Apparently this idea is valid only when we can suppose that the percentage of sold and retired investments in net investments is substantially small and almost neglisible. We call this “Zero way,” which has been adopted by Hori, Saito, and Ando (2004). In this case, sold and retired investments should be considered as a part of depreciated capital stocks. However, non-periodic lumpy disinvestments are never captured.

Zero way tracks the dynamics of only purchased investments, but the other two track the dynamics of net investments, which contain not only purchased investments but also sold and retired investments.

3.2 Sample Periods and Outliers

We construct the data-set of investments and capital stocks from FY 1978 to FY 2010. However, we exclude the samples before 1982 from the scope of our analysis since they may be strongly biased by the benchmark year effect. We estimate the model for the entire sample period (FY1982 to FY2010) as well as for the five sub-periods.[11] The spans of the five sub-periods are not the same because we divide them considering the underlying situations of Japanese economy such as the phase of business cycle.[12]

(1) The 1st period:	FY 1982–FY 1986	(Before the Bubble Economy)
(2) The 2nd period:	FY 1987–FY 1991	(The Bubble Economy)
(3) The 3rd period:	FY 1992–FY 1997	(After the Bubble Economy)
(4) The 4th period:	FY 1998–FY 2002	(Banking Crisis)
(5) The 5th period:	FY 2003–FY 2010	(The 14th Business Cycle)

[11] The basic idea of sample period devision is following Tonogi, Nakamura, and Asako (2010).

[12] We also conduct factor analysis by each fiscal year. The main results, which are not reported here, do not change much. They are available upon request.

It is important to remove the outliers for calculating the average of each investment rate in each account year robustly to subtract the trend from the actual investment rate.[13] The investment rates of 0.5% upper and lower tail (1% in total) of each fiscal year are removed to avoid extraordinary reductions and acquisitions of capital stocks pertaining to M&A and privatization of publicly owned companies. We also remove the data whose total assets at book value increased or decreased by more than 50%. To focus our analysis on the investment in tangibles, the data whose Tobin's q[14] values are at the 0.5% upper and lower tail (1% in total) are also removed. We do this because intangible-intensive firms like information and communication technology firms tend to have large Tobin's q values as the denominators are small and virtually close to zero. Finally, we remove the data whose debt ratios are on the 1% upper tail because we consider that these firms determine their investments in unusual ways owing to their critical financial situations. For further details about the data, Tonogi, Nakamura, and Asako (2010) and Asako and Tonogi (2010) may be referred to.

3.3 Estimation Model

We estimate the following factor model:

$$\boldsymbol{Z} = \boldsymbol{F}\boldsymbol{A}' + \boldsymbol{U}\boldsymbol{D}$$

where $\boldsymbol{Z} = [z_{ij}]$ is the investment rate of firm i in capital stock j, $\boldsymbol{F} = [f_{iq}]$ is a q^{th} common shock among all the investment rates of firm i, and $\boldsymbol{A} = [a_{jq}]$ is a factor loading that indicates a reaction of investment in capital stock j to the q^{th} common shock and applies uniformly to all firms. Then we decompose a correlation matrix $\boldsymbol{R}$ of the investment rates into communality $\boldsymbol{A}\boldsymbol{A}'$ and uniqueness $\boldsymbol{D}'\boldsymbol{D}$,

[13] The reason why this is important is described in 3.3.

[14] We define Tobin's q as (Market value of the firm less values of assets other than capital stock)/Repurchasing value of capital stock.

$$\boldsymbol{R} = \boldsymbol{A}\boldsymbol{A}' + \boldsymbol{D}'\boldsymbol{D}.$$

In ordinary multivariate statistical analysis, it is assumed that samples are generated randomly and independently from a multivariate normal distribution. In our analysis, we treat the investment rates in a similar manner. If we perform factor analysis to cross-section samples at each fiscal year, a macro trend of each investment rate is already removed as an average among firms in the correlation matrix. However, we conduct factor analysis for the entire sample period as well as for the five sub-periods. In these cases, we have to pay attention to the macro trends, which are likely autocorrelated. From the above, we calculate the averages of investment rates among firms at each period as their trends and subtract them from the corresponding investment rates before conducting factor analysis.

Tonogi, Nakamura, and Asako (2010) found that the averages at each fiscal year were affected by large negative investment rates, where investment rates are defined as investments divided by total capital stock after the investments.[15] This point is clearly shown in the summary statistics in Table 1. Therefore, we also calculate the medians of the investment rates as trends to check the robustness of our results. The main results, which are not reported in this paper, did not change significantly.[16]

3.4 Empirical Results

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[15] The investment rate here is defined as $\frac{I}{(1-\delta)K'}$ and takes a value within the range of $-\infty$ to $\frac{1}{1-\delta}$.

This representation comes from a maximization problem: $V(A,K) = max_I\ [AK^{\alpha} - \frac{\gamma}{2}\left(\frac{I}{(1-\delta)K'}\right)^2 (1-\delta)K' - pI + \beta E_{A'|A}\{V(A',K')\}]$,where $I = K' - (1-\delta)K$, which follows the well-known work of Hubbard (1998). In this case, it is supposed that invested capital immediately contributes to production, while invested capital are supposed to start the operation from the next period in models appeared in 2.1. It is easy to show that $I/(1-\delta)K'$ has a linear function of $V/(1-\delta)K$ in contrast to (2) and (3).

[16] These results are available upon request.

There are two important points to bear in mind while interpreting the empirical results of factor analysis:

1. Vector distances of factor loadings among investment rates.

2. The uniqueness of each investment rate.

Regarding point 1, if some investment rates have similar factor loadings, it implies that the parameters for adjustment costs of these investments would also be similar in theoretical modeling, although we do not exactly identify the functional structure of the model. With respect to point 2, if some investment rates have low values of uniqueness, it suggests that theoretical models can explain these investment rates fairly well. We interpret our estimation results based on these two points. We also take a close look at how different the results are depending on data construction methods, industries, and sample periods discussed in 3.2.

Table 2 shows the results of factor analysis on the data of entire sample period from FY 1982 to FY 2010, in which the average of each investment rate by fiscal year is subtracted from the actual investment rate as its trend. The table contains the results of "Proportional way," which estimates the current values of sold and retired investments, of "Book-value way," where current values of sold and retired investments are replaced with their book values, and of "Zero way," where sold and retired investments are igonored or assumed to be zero. Based on Table 2, Figure 2 depicts vector distances (squared) between the investment rates in common factor space.

From the viewpoint of factor loading similarities, it is conspicuous that the vector distance between [1] Building and [2] Structure is close to zero. These investment rates have the largest factor loadings to the first factor, as shown in Table 2. Separately, we also find that the vector distances between [3] Machinery & Equipment, [4] Vehicles & Delivery Equipment, and [6] Tools, Furniture, & Fixture are less than 0.05. These investment rates have the largest factor loadings to the second factor, as shown in Table 2. The third factor has nothing more than a trivial impact on all the investment rates because of the small value of their factor loadings. Incidentally, the factor loading of [5] Shipment is largest among all the investment rates in each way of data construction.

From the above, even though we do not know the functional structures of the adjustment costs of the investment rates, it is implied that [1] Building and [2] Structure have similar parameters for their adjustment costs and separately that [3] Machinery & Equipment, [4] Vehicles & Delivery Equipment, and [6] Tools, Furniture, & Fixture have similar parameters for their adjustment costs. It is worth highlighting that the vector distances between [7] Land and [1] Building and between [7] Land and [2] Structure are not close to zero, while almost all researchers are likely to agree that these three investment rates might be influenced strongly by the same factor because buildings and structures are constructed on land. We consider this gap as the evidence that [7] Land has been playing the roles other than a factor of production such as a collateral and a tool of asset management for the listed Japanese firms.

To alleviate the Curse of Dimensionality, it would be useful to bundle investments together when their vector distances are close to zero. The differences between the group of investment goods that reacts strongly to the first factor and the other group that reacts to the second factor might arise in their depreciation schedules, the ease of resale, and the time required for installation during which operations are shut down. We could express these characteristics by the difference in parameter values of the irreversibility of investments, of the asymmetry in purchase and resale prices, and of fixed costs of investment with the opportunity costs of a shutdown. It would be important to compare the parameter values of adjustment costs between these groups. We plan to address this issue in future studies.

In Table 2 and Figure 4(1), we find the following relationship among the uniqueness of the investment rates, which represents the percentages of their variances not explained by common shocks.

Building $<$ Structure $<$ Machinery & Equipment $<$ Tools, Furniture & Fixture

$<$ Vehicles & Delivery Equipment $<$ Land $<$ Shipment.

In particular, the values of uniqueness of [1] Building and [2] Structure based on Zero way are about 0.5 and those based on Proportional way are about 0.7. It is reasonable to say that a major part of the dynamics of these investment rates can be replicated by theoretical models. On the other hand, the values of uniqueness of [5] Shipment and [7] Land are about 0.9 regardless of the way of capital stock data construction. Those variances are considered to be originated from firm-specific individual factors.

It is noteworthy that Tonogi, Nakamura, and Asako (2010), which applied the Multiple q model with convex adjustment costs to almost the same dataset as the present paper, found that R^2 values were not much more than 0.1 in all cases. Taking the sufficiently low values of uniquness into account, we have much room to improve the tractability of the data under Multiple q frameworks with comprehensive adjustment costs.

When we look at the differences in the values of uniqueness resulting from the differences in data construction ways of capital stocks, we find the following relationship:

Proportional way > Book-value way > Zero way

This might be caused not only by the differences in errors of estimates for sold and retired investments, but also by the differences in behaviors of purchased investments and sold and retired investments.[17] However, these issues are beyond the scope of the present paper.

Figure 3 and Figure 4-(2) show the results of factor analysis on manufacturing firms. When we apply the factor analysis to all industries, the estimated factor loadings tract the reactions to the common factors among all industries and the difference between the industries are left in the individual factors. Therefore, we can expect that the estimated values of uniqueness are higher than those of manufacturing firms. However, Figure 4 indicates that the values of uniqueness of all industries are only slightly higher than those of manufacturing, which implies that the differences in investment dynamics between manufacturing and service industries are trivial.

When we take a closer look at Figure 4, the values of uniqueness for [3] Machinery & Equipment take substantially lower values in manufacturing than all industries especially under Book-value way and Zero way, while those of [6] Tools, Furniture, & Fixture do not take regardless of the similarity in factor loadings with [3] Machinery & Equipment. These issues are also worth noting but beyond the scope of this paper.

Figure 5 and Figure 6 show our results for each of the five sub-periods, which do not reveal remarkable differences compared to the results of entire period (Table 2). From the viepoint

[17] Zero way tracks the dynamics of purchased investments only, while other ways track the dynamics of both purchased investments and sold and retired investments.

of parameter values and the explanatory power of underlying theoretical models, we can say that the differences among periods as well as among industries are not so significant. Interestingly, however, the figures also show that the patterns in time-series transition of factor loadings and uniqueness are similar between Book-value way and Zero way, while those of Proportional way is quite different from the others. Also in this respect, the dynamics of sold and retired investments are worth investigating in our future work .

4. Conclusions

In this paper, we examined the heterogeneity of capital stocks using financial statement data of listed Japanese firms. We conducted factor analysis on the investment rates of seven capital stocks and estimated their factor loadings as reactions to common factors (corresponding to TFP shocks). If some of the estimated factor loadings are similar, it implies that the parameters for the adjustment costs of these investments are also similar even though we do not specify the functional structures of these costs for each investment. Simultaneously, we decomposed the variance for each investment rate into communality, the percentage of variance that is explained by the common factors, and uniqueness, the percentage of variance that is not explained by the common factors. If the values of uniqueness for some investment rates are low, it implies that theoretical models can track the dynamics of these investment rates fairly well.

We first implemented the factor analysis on the entire data set from FY 1982 to FY 2010. Then we divided our data into five sub-periods. In advance of these estimations, we calculated the averages of investment rates at each period and subtracted them from actual investment rates as their trends to avoid the impact of autocorrelation through the trends. We also conducted the same analyses on manufacturing firms to gauge the difference between them and service-oriented firms.

Our results showed that [1] Building and [2] Structure had similar factor loadings, while those for [3] Machinery & Equipment, [4] Vehicles & Delivery Equipment, and [6] Tools, Furniture, & Fixture were similar. Grouping together the investments with similar factor loadings can remedy the Curse of Dimensionality. We also found that the values of uniqueness of [1] Building and [2] Structure were substantially low, which suggests that it is possible to track these investment rates using an Multiple q model with comprehensive adjustment costs. We will attempt estimating the structural models in future studies.

To explore the differences between two groups of investment further, we should take a closer look at characteristics such as depreciation schedules, the ease of resale, and the duration of shutdown required for installation. These characteristics are considered to be captured as differences in the parameter values of adjustment cost. It would be also useful to estimate and compare these parameter values between two groups in our future studies.

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Table 1: Summary Statistics

(1) Proportional way

	Obs	Mean	Std. Dev.	Min	Max
Building	58154	0.0540	0.32	−10.50	0.90
Structure	58124	0.0509	0.40	−31.37	1.06
Machinery & Equipment	58083	0.0753	0.39	−16.25	1.10
Shipment	58014	0.0020	0.04	−1.60	1.16
Vehicles & Delivery Equipment	58085	0.0245	1.05	−42.65	1.17
Tools, Furniture & Fixture	58148	0.0972	0.50	−25.39	0.94
Land	58172	0.0040	0.30	−9.80	1.03

(2) Book-value way

	Obs	Mean	Std. Dev.	Min	Max
Building	58204	0.0751	0.13	−2.16	0.88
Structure	58151	0.0793	0.14	−1.72	1.06
Machinery & Equipment	58108	0.1062	0.13	−1.22	1.10
Shipment	58024	0.0022	0.04	−1.32	1.15
Vehicles & Delivery Equipment	58118	0.1262	0.19	−1.67	1.17
Tools, Furniture & Fixture	58207	0.1453	0.11	−0.64	0.86
Land	58180	0.0043	0.30	−9.80	1.03

(3) Zero way

	Obs	Mean	Std. Dev.	Min	Max
Building	58462	0.0839	0.11	0.00	0.89
Structure	58411	0.0857	0.12	0.00	0.98
Machinery & Equipment	58383	0.1117	0.12	0.00	1.10
Shipment	58321	0.0028	0.03	0.00	1.09
Vehicles & Delivery Equipment	58391	0.1397	0.15	0.00	1.11
Tools, Furniture & Fixture	58485	0.1480	0.11	0.00	0.88
Land	58442	0.0406	0.10	0.00	0.93

(Note) These are summary statistics for investment rates ($= I_j/(1-\delta)K_j'$) of each capital stock from FY 1982 to FY 2010 after removing the outliers. As for details about how to deal the outliers, refer to section 3.2.

Table 2: Factor Loadings and Uniqueness (All Periods, All Industries)

(1) Proportional way

	Factor 1	Factor 2	Factor 3	Uniqueness
Building	0.463	0.033	0.001	0.78
Structure	0.397	0.061	−0.001	0.84
Machinery & Equipment	0.271	0.196	0.002	0.89
Shipment	−0.004	0.028	0.019	1.00
Vehicles & Delivery Equipment	0.060	0.155	0.002	0.97
Tools, Furniture & Fixture	0.239	0.202	0.000	0.90
Land	0.250	−0.092	0.002	0.93

(2) Book-value way

	Factor 1	Factor 2	Factor 3	Uniqueness
Building	0.695	0.033	0.002	0.52
Structure	0.635	0.048	0.000	0.59
Machinery & Equipment	0.408	0.226	0.002	0.78
Shipment	−0.003	0.016	0.054	1.00
Vehicles & Delivery Equipment	0.201	0.249	0.003	0.90
Tools, Furniture & Fixture	0.424	0.202	−0.010	0.78
Land	0.224	−0.071	0.012	0.94

(3) Zero way

	Factor 1	Factor 2	Factor 3	Uniqueness
Building	0.720	0.071	−0.002	0.48
Structure	0.671	0.066	0.002	0.55
Machinery & Equipment	0.400	0.271	0.001	0.77
Shipment	−0.006	−0.028	0.071	0.99
Vehicles & Delivery Equipment	0.230	0.299	0.004	0.86
Tools, Furniture & Fixture	0.438	0.255	−0.018	0.74
Land	0.282	0.011	−0.003	0.92

(Note) These are the results of the factor analysis oninvestment rates of each capital stock after removing their trends, which are calculated by averages of each fiscal year. We implemented the principal factor method and orthogonal varimax rotations.

Figure 1: Image of Factor Analysis (1 Factor, 2 Capital Stocks)

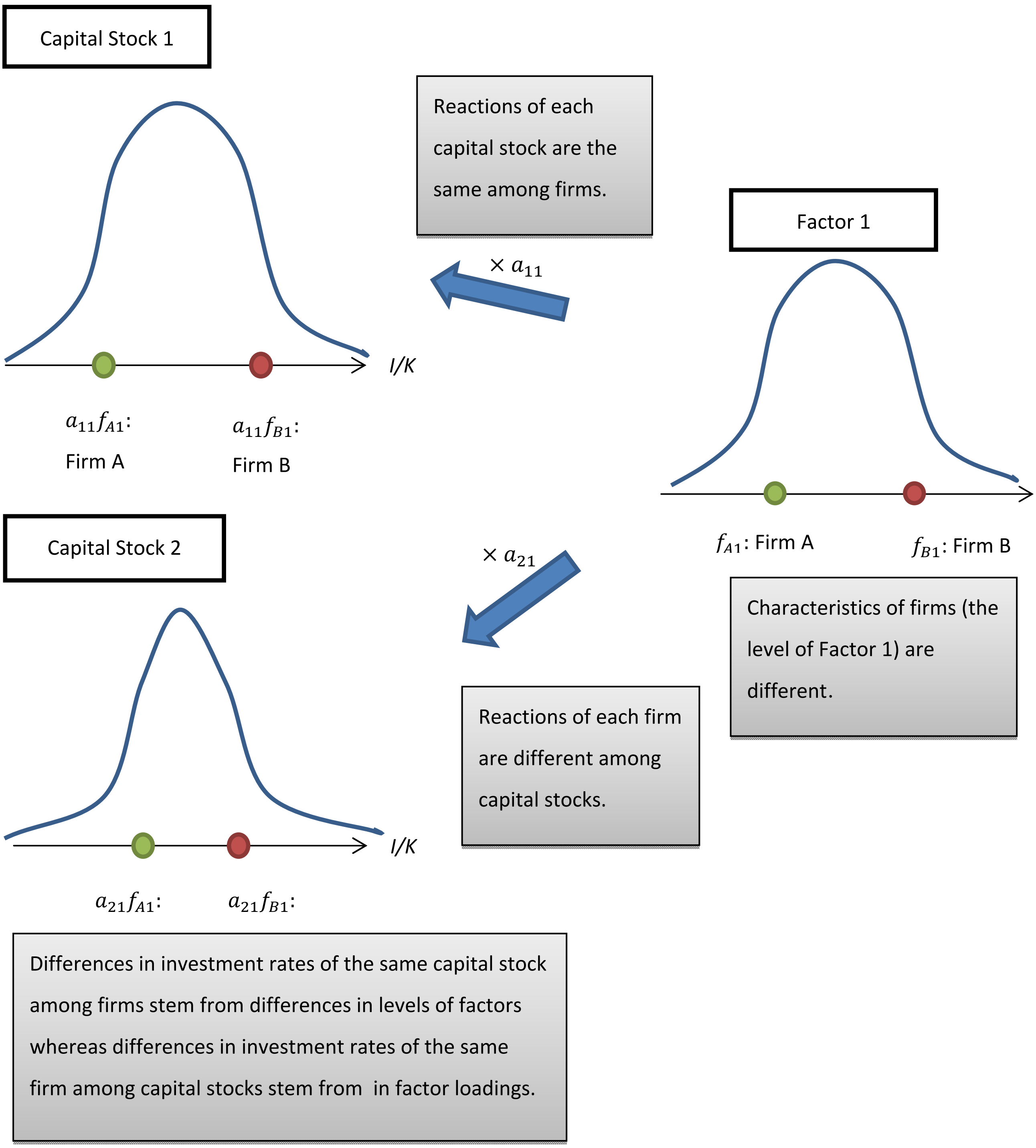
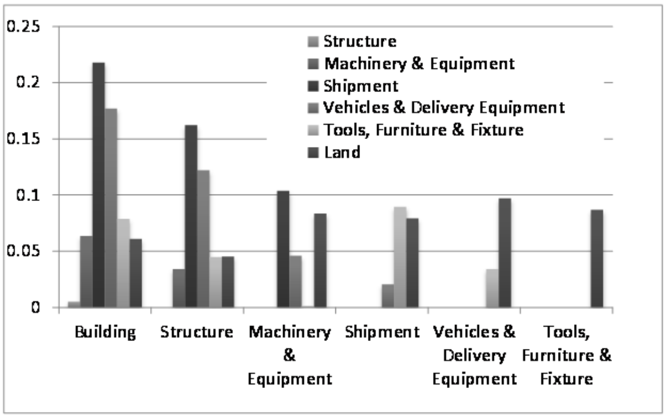
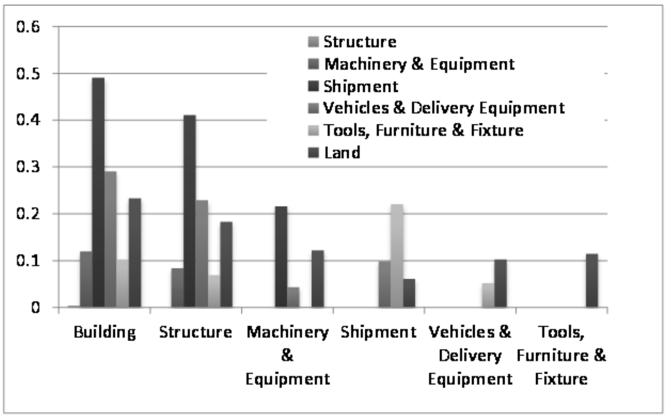


Figure 2: Distances between Vectors of Factor Loadings in Common Factor Space (All Periods, All Industries)

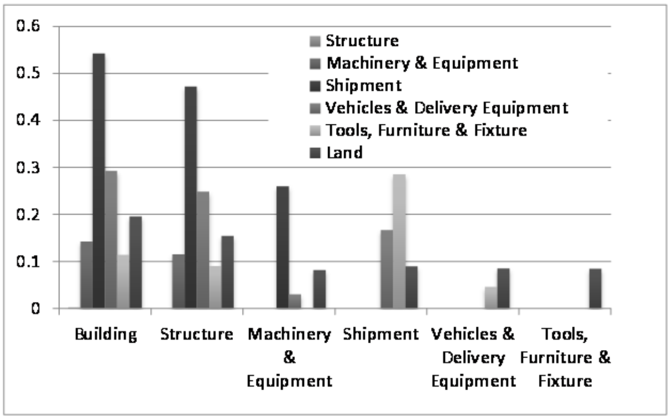
(1) Proportional way



(2) Book-value way



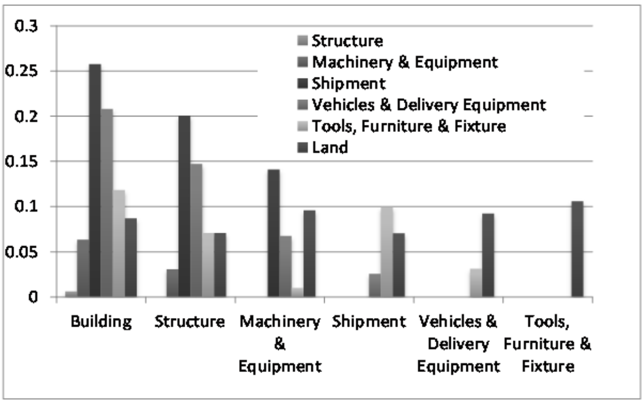
(3) Zero way



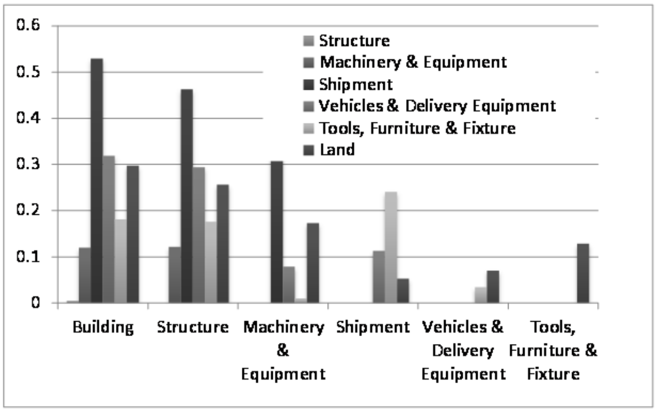
(Note) The vertical axis indicates the distances (squared) between the factor loading vectors (shown in Tale 2) of two investment goods. After omitting overlapping results, $21(= {}_7C_2)$ distances of each pair of goods appear in a panel. For example, the distance between Building and Structure is shown at the item name "Structure" on the horizontal axis.

Figure 3: Distances between Vectors of Factor Loadings in Common Factor Space (All Periods, Manufacturing)

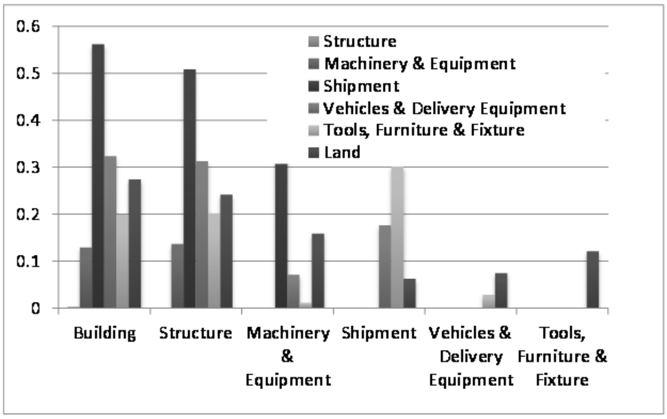
(1) Proportional way



(2) Book-value way



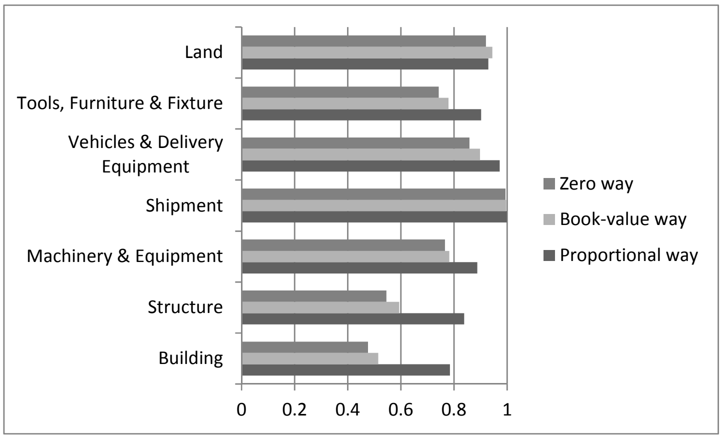
(3) Zero way



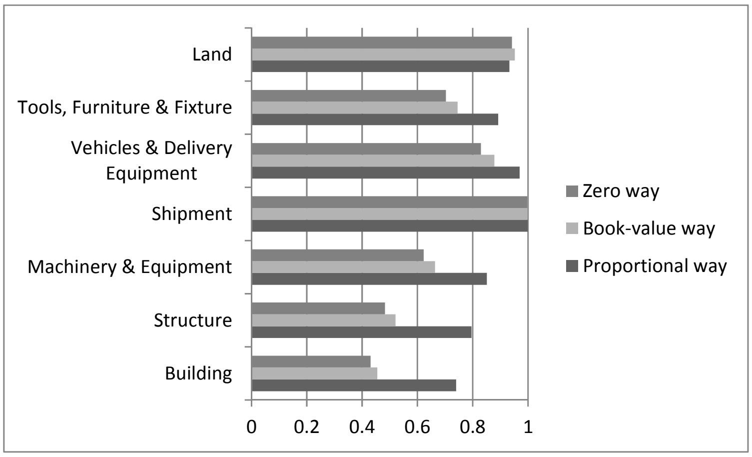
(Note) See the note of Figure 2.

Figure 4: Uniqueness (All Periods)

(1) All industries



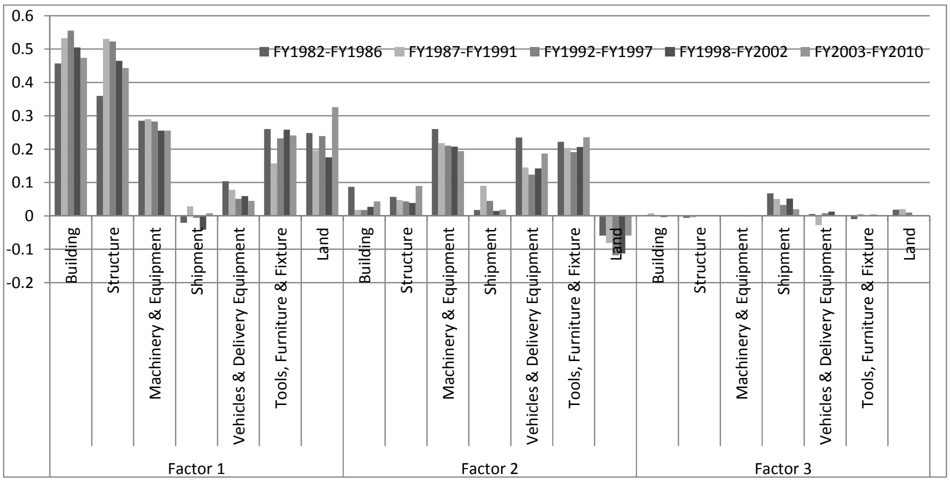
(2) Manufacturing



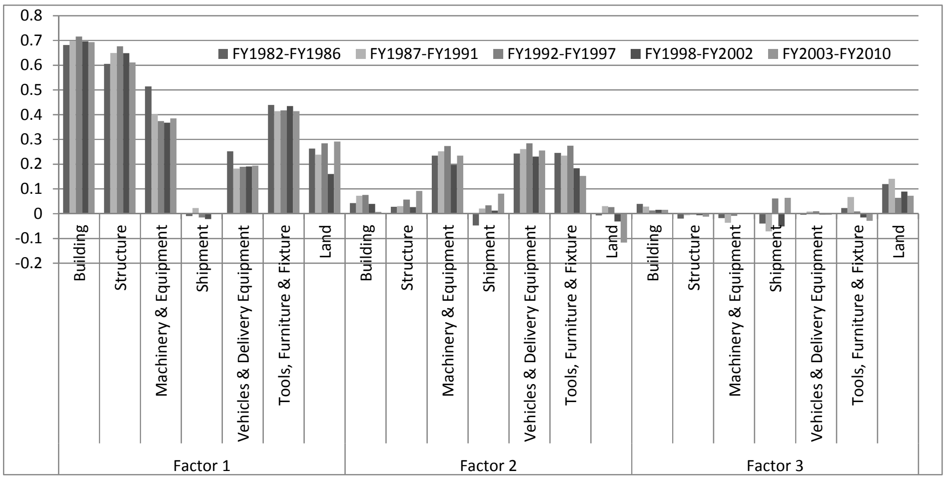
(Note) Figure 4-(1) is based on the same estimation results as Table 2 and Figure 2. Figure 4-(2) is based on the same estimation results as Figure 3.

Figure 5: Factor Loadings (Each Period, All Industries)

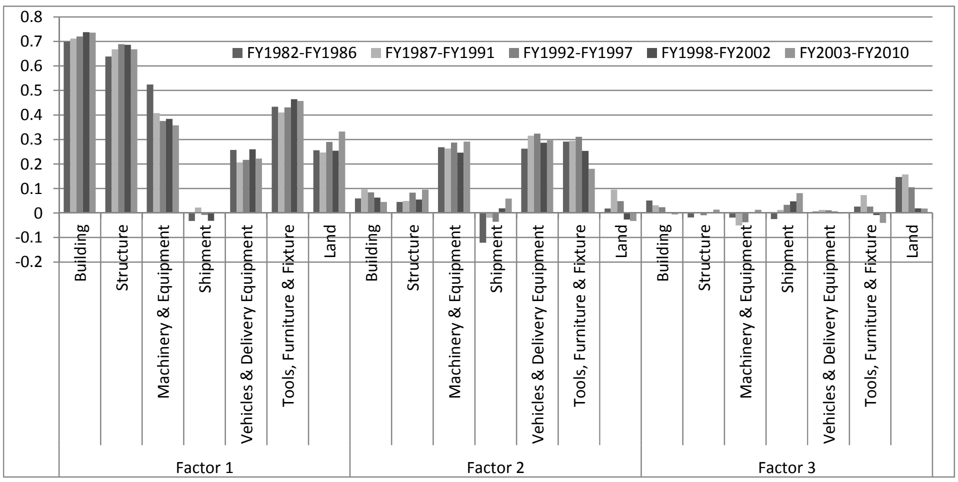
(1) Proportional way



(2) Book-value way



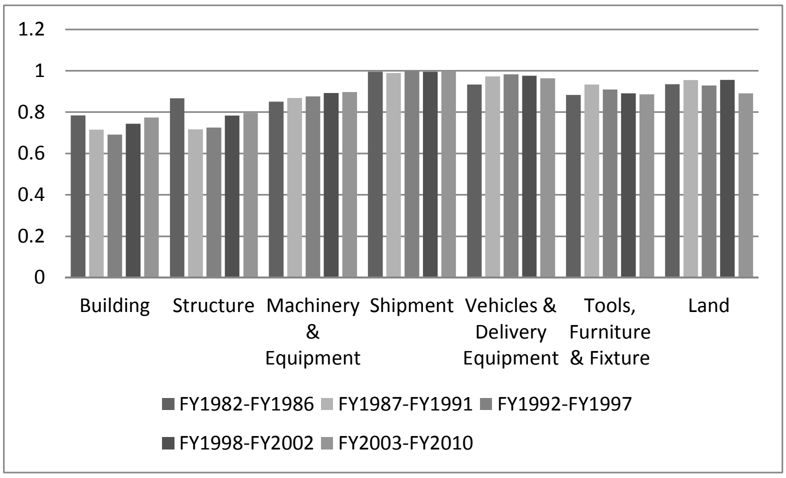
(3) Zero way



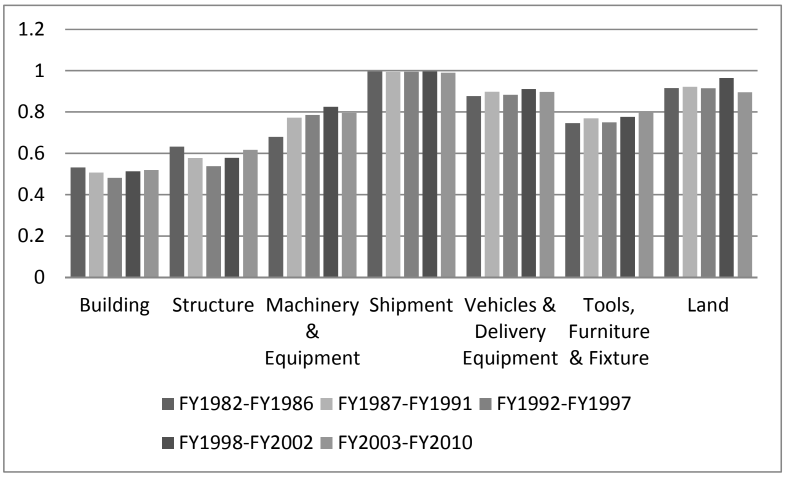
(Note) These figures show the results of factor analysis on five sub-periods.

Figure 6: Uniqueness (Each Period, All Industries)

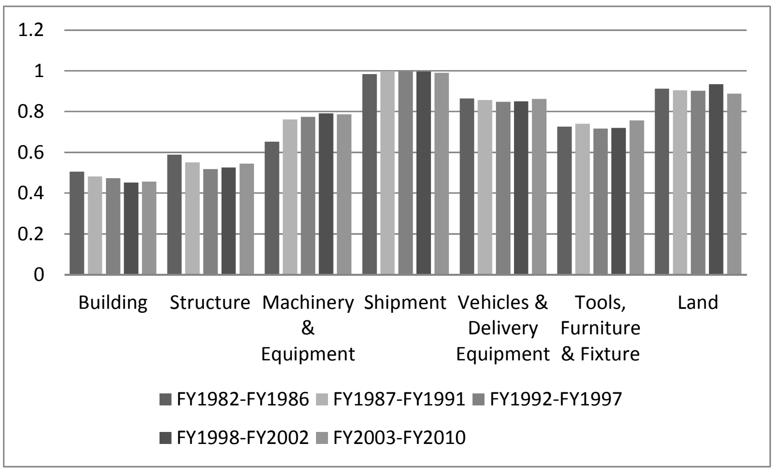
(1) Proportional way



(2) Book-value way



(3) Zero way



(Note) These figures are based on the same estimation results of factor analysis as Figure 5.

Appendix: The relationship between investment equations and factor models

In this Appendix, we show the relationship between equation (5) [18]and (8), i.e.,

$$\begin{aligned}\frac{I_j}{K_j} &= \frac{1}{\gamma_j}\left(\beta E_{A'|A}\left[\partial V(A', K_1', \cdots, K_n')/\partial K_j'\right] - p_j\right) \\ &= \frac{1}{\gamma_j}(q_j - 1)p_j \quad \text{where } q_j = \frac{\beta E_{A'|A}[V_{K_j}(A', K_1', \cdots, K_n')]}{p_j},\end{aligned} \tag{5}$$

and

$$z_{ij} = a_{j1}f_{i1} + a_{j2}f_{i2} + \cdots + a_{jm}f_{im} + d_j u_{ij}\,. \tag{8}$$

We obtain z_{ij} by standardizing I_j/K_j:

$$z_{ij} = \frac{I_j/K_j - E[I_j/K_j]}{\sqrt{\mathrm{Var}(I_j/K_j)}} = \frac{\frac{1}{\gamma_j}(q_{ij} - 1)p_j - \frac{1}{\gamma_j}E[(q_{ij} - 1)p_j]}{\sqrt{\mathrm{Var}(I_j/K_j)}} = \frac{\overline{\overline{(q_{ij} - 1)p_j}}}{\gamma_j\sqrt{\mathrm{Var}(I_j/K_j)}}$$

where variables with double lines represent the deviations from their means among firms and q_j is derived by the derivative of the value function. Then since $q_1, \cdots, q_n$ are all driven by the same A', $(q_j - 1)p_j$ can be linearly approximated by

$$\overline{\overline{(q_{ij} - 1)p_j}} \cong \rho_{j1}\overline{\overline{EA'_1}} + \rho_{j2}\overline{\overline{EA'_2}} + \cdots + \rho_{jm}\overline{\overline{EA'_m}} + \epsilon_{ij}\,. \tag{A1}$$

where EA'_s $(s = 1, \cdots, m)$ are the components of the expectation of the next period's TFP, $E_{A'|A}[A']$, and ϵ_{ij} is the approximation error. Note that the errors may arise from the differences in capital stock $K_j (j = 1, \cdots, n)$ among firms. Then we obtain

[18] We can add the measurement error in each investment rate. In this case, $\mathrm{Var}(I_j/K_j)$ increases by the variance of the error and z_{ij} is influenced by the error.

$$z_{ij} = \frac{\rho_{j1}\overline{\overline{EA'_1}} + \rho_{j2}\overline{\overline{EA'_2}} + \cdots + \rho_{jm}\overline{\overline{A'_m}} + \epsilon_{ij}}{\gamma_j\sqrt{\mathrm{Var}(I_j/K_j)}}.$$

The factors $f_s(s = 1,2, \cdots, m)$ are assumed that their variances equal 1 in equation (11). Then the relationship between the factors and the TFP is as follows:

$$f_s = \frac{\overline{\overline{EA'_s}}}{\sqrt{\mathrm{Var}(EA'_s)}} . \tag{A2}$$

From the above, the factor loadings satisfy

$$a_{js} = \frac{\rho_{js}\sqrt{\mathrm{Var}(EA'_s)}}{\gamma_j\sqrt{\mathrm{Var}(I_j/K_j)}} \qquad (s = 1,2, \cdots, m). \tag{A3}$$

The individual factors are derived by

$$d_j u_{ij} = \frac{\epsilon_{ij}}{\gamma_j\sqrt{\mathrm{Var}(I_j/K_j)}} . \tag{A4}$$

From (A1) to (A4), the factors correspond to Partial q driven by TFP shocks, the factor loadings correspond to the parameters of adjustment costs, and individual factors are composed of approximation errors, which contain the differences in capital stock $K_j(j = 1, \cdots, n)$ among firms. If some of the investments are homogenous goods and have similar values of ρ_{js} $(s = 1, \cdots, m)$ and γ_j and their variances[19], then the factor loadings of these investments are also similar.

[19] This means that they have the same functional structures in a theoretical model.

If non-convex adjustment costs are introduced in profit-maximizing models, the dynamics of investment rates becomes nonlinear (lumpy and discontinuous). In such a scenario, linear approximation might be inappropriate. Suppose that an investment equation can be written as

$$\frac{I_j}{K_j} = \frac{1}{\lambda_j}\phi_j(EA', K_1', K_2', \cdots, K_n{}')$$

where $\phi_j(\cdot)$ is a nonlinear function with a variance of 1 and that $\phi_j(\cdot)$ can be decomposed as

$$\overline{\overline{\phi_J(EA', K_1', K_2', \cdots, K_n')}} = \overline{\overline{\phi_{J1}(EA_1{}')}} + \overline{\overline{\phi_{J2}(EA_2{}')}} + \cdots + \overline{\overline{\phi_{Jm}(EA_m{}')}} + \epsilon_{ij} \quad \text{(A5)}$$

where $\phi_{js}(\cdot)$ $(s = 1, \cdots, m)$ are also nonlinear functions and independent of each other.

Then we obtain

$$f_{js} = \overline{\overline{\phi_{Js}(EA_s{}')}}$$
$$a_{js} = \frac{\sqrt{\phi_{js}(EA_1')}}{\lambda_j\sqrt{\mathrm{Var}(I_j/K_j)}}$$

These are the nonlinear version of (A2) and (A3).